International Journal of Modern Physics B
Vol. 23, Nos. 12 & 13 (2009) 2948–2954

VALLEY SPLITTING AND g-FACTOR IN AlAs QUANTUM WELLS

C. A. DUARTE

Departamento de Física, Universidade Federal do Paraná,
CP 19044, 81531-990–Curitiba-PR, Brazil
celso@fisica.ufpr.br

G. M. GUSEV, T. E. LAMAS and A. K. BAKAROV*

Instituto de Física da Universidade de São Paulo,
CP 66318, 05314-970–São Paulo-SP, Brazil

J.-C. PORTAL

GHMFL-CNRS, Boite Postale 166, F-38042 Grenoble Cedex 9, France
INSA-Toulouse, 31077 Cedex 4, France;
and Institut Universitaire de France, Toulouse, France

Received 25 July 2008

Here we present the results of magneto resistance measurements in tilted magnetic field and compare them with calculations. The comparison between calculated and measured spectra for the case of perpendicular fields enable us to estimate the dependence of the valley splitting as a function of the magnetic field and the total Landé g-factor (which is assumed to be independent of the magnetic field). Since both the exchange contribution to the Zeeman splitting as well as the valley splitting are properties associated with the 2D quantum confinement, they depend only on the perpendicular component of the magnetic field, while the bare Zeeman splitting depends on the total magnetic field. This information aided by the comparison between experimental and calculated gray scale maps permits to obtain separately the values of the exchange and the bare contribution to the g-factor.

Keywords: AlAs; valley splitting; g-factor.

1. Introduction

The conduction band in AlAs has two minima which originate from six equivalent valleys located in the Brillouin zone close to the X points, where four of them are shifted up by quantum confinement. The remaining two are populated when the structure is n-doped. As a consequence, the Landau level (LL) diagram is four fold

*Permanent Address: Institute of Semiconductor Physics, Novosibirsk, 630090 Russia.

split, since each LL corresponds to two spin split X-valley (VS) levels. It is well known that VS, that occurs not only in AlAs but also for example in Si, can be caused by other mechanisms, as for example, strain; however, strong magnetic fields perpendicular to a two dimensional electron gas (2DEG) confined in such materials structures are known to enhance VS, and on the other side in-plane fields do not have influence.[1] This leads us to believe that this enhancement is associated to a property of the 2DEG, and so is associated with electron-electron interactions, what has been considered controversial.[2]

Previous work[1] studied n-doped AlAs/GaAs quantum wells of different widths where the electron density was controlled by illumination and by the application of gate voltages. As a result, it was found that the energy gap E_{VS} between the two lower energy valley split bands follows a linear dependence with respect to the applied (perpendicular component of) magnetic field, at least in the range of field between 2 T - 4 T, and below this range the linearity is lost, the behavior is in any case monotonic. In this work, the values of E_{VS} were extracted using the coincidence method,[3] in which a series of spectra took at successive tilt angles is used to estimate the values of E_{VS} with respect to the size of cyclotron gap $\hbar\omega_c$ in the crossing points of different Landau levels (LL). It is worth to note that this method can also be used, of course, to estimate the Zeeman energy, and this was also done in the mentioned paper.

In the present work we estimate E_{VS} and the Zeeman gap comparing the experimental spectra to the ones obtained by calculation, using adequate fitting parameters. We found the dependence of the VS gap in the range of 0.6 T to 6 T and found a nonlinear behavior.

2. Description of Samples and Measurements

The studied samples were modulation Si doped AlAs square quantum wells 150 Å wide, grown on GaAs (100) substrates by molecular beam epitaxy. Characteristic growth parameters are presented on Table 1.

Table 1. Growth parameters of AlAs samples.

Sample number	Width (Å)	Si doping (cm^{-2})	Spacer (bottom/top)	As source
3237	150	1.5×10^{18}	300/350 Å	As_2
3238	150	1.2×10^{18}	SL[a]	As_2
3239	150	same as 3237	same as 3237	As_4
3240	150	same as 3238	same as 3238	As_4

[a]Superlattice: bottom: 31 Å AlGaAs+{5ML(AlAs)+6ML(GaAs)}×10;
top: 31 Å AlGaAs+{5ML(AlAs) + 6ML(GaAs)}×9.

After growth, we prepared Hall bars 500×200 μm wide by conventional lithography. We performed Shubnikov-de Haas measurements at ^{3}He temperatures on a closed cycle refrigerator (≈300mK) from zero up to 15 T, varying the tilt angle of the sample with respect to the applied magnetic field.

Samples 3237, 3239 and 3249 showed lower mobility, and the best spectra were obtained from sample 3238, for which the data are presented in the following sections.

3. Theoretical Background

Longitudinal magneto resistance can be simulated theoretically using a simple model based on the occupation of the successive LL's by the 2DEG confined in the structure, where for simplicity we do not consider the fraction of delocalized states. We start from a model density of states (DOS) consisting of Lorentzian shaped peaks each one centered on each LL. Since each LL is split by spin and valley splitting, we must consider this four fold spanning and write the DOS as a function of the energy E, the magnetic field B and its perpendicular component $B_\perp$:[2,4,5]

$$DOS(E,B) = \frac{eB_\perp}{h} \sum_{\nu} \sum_{s=+,-} \sum_{N=0}^{\infty} \Gamma_{N,s,\nu} \left(1 + \left(\frac{E_{N,\nu,s}(B) - E}{\Gamma_{N,s,\nu}}\right)^2\right)^{-1} . \tag{1}$$

In this expression, N and s are the LL and spin indices, and ν is the VS index which is associated with the two – the upper and the lower – energy values. Then $E_{N,s,\nu}(B)$ is the energy level associated with the LL/spin/VS indices $N/s/\nu$, respectively. $\Gamma_{N,s,\nu}$ is the corresponding level broadening, which is assumed to be equal for all levels, i.e., $\Gamma_{N,s,\nu} = \Gamma = \text{constant} = h/\tau_q$,[2,4] where τ_q is the quantum lifetime. With this we can calculate the energy of the Fermi level E_F solving the following integral equation, which depends on the total electron sheet density n_s:

$$n_s = \int_{-\infty}^{E_F(B)} DOS(E,B)dE. \tag{2}$$

When we have solved this equation, we have $E_F(B)$. Then we may calculate the longitudinal conductivity by the following expression derived by the Kubo formula:[5]

$$\sigma_{xx}(B) = \frac{e^2}{h} \sum_{\nu=1,2} \sum_{s=+,-} \sum_{N=0}^{\infty} \left(N + \frac{1}{2}\right) \left(1 + 2\left(\frac{E_F - E_{N,\nu,s}}{\Gamma}\right)^2\right)^{-1} . \tag{3}$$

Finally, we get the longitudinal magneto resistance ρ_{xx} by the inversion of the conductivity tensor $\widetilde{\sigma}$:

$$\rho_{xx} = \sigma_{xx} / \left(\sigma_{xx}^2 + \sigma_{xy}^2\right) , \tag{4}$$

where we can use the usual approximation for Hall conductivity $\sigma_{xy} = n_s e / B_\perp$.

The next necessary step for our analysis is the inclusion of the dependence of valley and spin splitting on the tilt angle θ. The energies that enter on Eq. 3 are given by

$$E_{N,\nu,s} = \left(N + \frac{1}{2}\right) \hbar\omega_c + E_{VS,\nu} + g^* \mu_B B, \tag{5}$$

where $E_{VS,\nu}$ are the energies of the two VS levels and $g^*\mu_B B$ is the Zeeman energy, which is written in terms of the effective total g-factor g^*, that is related to the bare g-factor g_0 and the exchange contribution energy E_{ex} by the following expression:

$$g^*\mu_B B = g_0\mu_B B + E_{ex}(B_\perp) . \tag{6}$$

The exchange energy is assumed to depend linearly on the cyclotron energy:[6,7]

$$E_{ex}(B_\perp) = \alpha\hbar\omega_c . \tag{7}$$

As a consequence, the bare Zeeman term depends on the total magnetic field, while the exchange contribution depends only on the perpendicular component of the magnetic field.

The remaining unknown term is $E_{VS,\nu}$, which has to be determined as a function of magnetic field. So we have three constant parameters m^*, g_0, α and the function $E_{VS,\nu}$ to adjust. In addition we have the electron sheet density n_s, which can be determined either by Hall measurements or used as an additional fitting parameter.

4. Results and Analysis

The first step is to analyze the simplest case of perpendicular magnetic field. We determined n_s by Shubnikov-de Haas oscillations and started trying the value 2.0 for the bare Zeeman g-factor g_0 and an effective mass $m^* = 0.46m_0$.[1,8] For $E_{VS,\nu}$ we can take guess values from Fig. 4 of Ref. 1, for which a linear fitting formula $\Delta E_{VS} = -0.22 + 0.25B_\perp$ was established, where $\Delta E_{VS} = E_{VS,1} - E_{VS,2}$. A careful comparative analysis of peak positions of experimental and calculated $\rho_{xx}(B)$, varying the guess parameter values α, g_0, $E_{VS,\nu}(B)$, is sufficient to determine the optimal values.

In Fig. 1a we present both the experimental and calculated spectra of the longitudinal magneto resistance R_{xx} for sample 3238. We used $n_s = 2.12 \times 10^{11}$ cm^{-2} and a phenomenological quantum lifetime τ_q=30 ps. The effective mass was taken as above, and we used g_0=1.9 and α=0.05. These choices for the spin parameters correspond to a total effective g-factor g^*=4.8.[a]

We can verify a good agreement between experimental and calculated spectra. In the main Fig. 1a, peaks of both spectra are centered at very close values of magnetic field. Similarly, in the low field region presented in the insert of Fig. 1a, the series of pairs or spin-split peaks follow a sequential progression in which the average peak center and valley position is very similar to both spectra.

To obtain the close similarity, we needed to use a magnetic field dependence of $E_{VS,\nu}$ as is presented in Fig. 1b. In this figure we show the total VS gap $\Delta E_{VS,\nu} = |E_{VS,2} - E_{VS,1}| = 2|E_{VS,1}| = 2|E_{VS,2}|$ as a function of the perpendicular component of the magnetic field $B_\perp$.

[a]In the older Ref. 8, an anomalously high value g^*=9.0 was reported, while in the most recent Ref. 1 g^*=3.82.

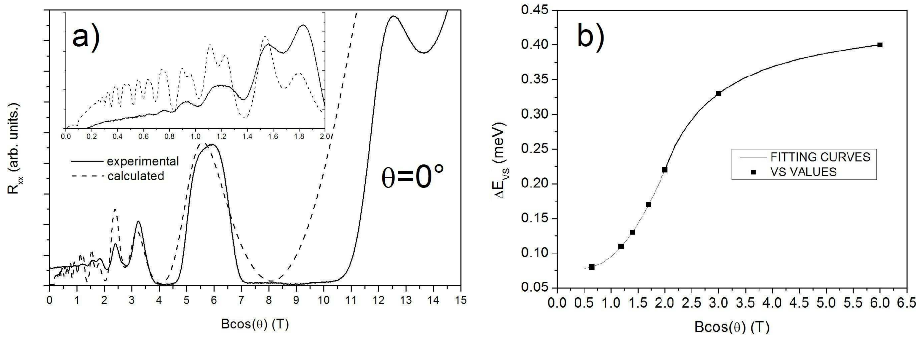

Fig. 1. a) Experimental (full line) and calculated (dashed line) spectra of the longitudinal magneto resistance R_{xx} as a function of magnetic field, in the case of perpendicular field (zero tilt angle, θ=0°). In the insert we detail the region of low magnetic field (from zero up to 2 T). b) $E_{VS,\nu}$ as a function of $B_\perp$. The squares mark the fitting values necessary to adjust successive peak centers. The full line represents the result of fitting polynomial and rational functions, respectively on the ranges 0.6 T – 2.0 T and 2.0 T – 6.0T.

The squares in Fig. 1b mark the values of $\Delta E_{VS,\nu}$ that were necessary to adjust the peak center. In other words, we tried successively increasing values for $\Delta E_{VS,\nu}$ and looked for the successive peak coincidences. Then, for example, the experimental peak at 6.0 T (Fig. 1a) required $\Delta E_{VS,\nu}$=0.4 meV for coincidence, and so on. After finishing the mapping of VS energy values (Fig. 1b, squares), we tried to fit them by curves. The chosen functions are presented in Eq. 7:

$$\Delta E_{VS} = \begin{cases} 0.09159 - 0.05705 B_\perp + 0.06063 B_\perp^2, & 0.6\ \mathrm{T} \le B_\perp \le 2.0\ \mathrm{T} \\ 0.44846 - 0.24604/\left(B_\perp - 0.92308\right), & 2.0\ \mathrm{T} \le B_\perp \le 6.0\ \mathrm{T}. \end{cases} \tag{8}$$

Now we are able to test the overall consistence of the parameters, trying to fit a set of spectra with variation of tilt angle. In Fig. 2 we show both the experimental (a) and the calculated (b) gray scale maps for the resistance, in the most significant range of magnetic fields. To construct the experimental map (a) from the experimental spectra, we subtracted a background classical magneto resistance and scaled each spectrum by a constant for each particular tilt angle, in order to make clear the topology of peaks in the full map. We verified that this procedure does not alter significantly the positions of the peaks, and does not invalidate neither our analysis, nor Fig. 1a and Eq. 8.

First of all, we note that in a wide range of tilt angles starting from zero there is a strong similarity between both diagrams, say, the predominance of almost vertical lines that tend to cross at angles above the center of the vertical scale. Also, there is similarity between the positions of the crossing points in both diagrams.

5. Conclusion

As a conclusion, we verified that the simple model for the magneto resistance presented in Sec. 3 is sufficient for our analysis. The fitted value for g_0 is consistent

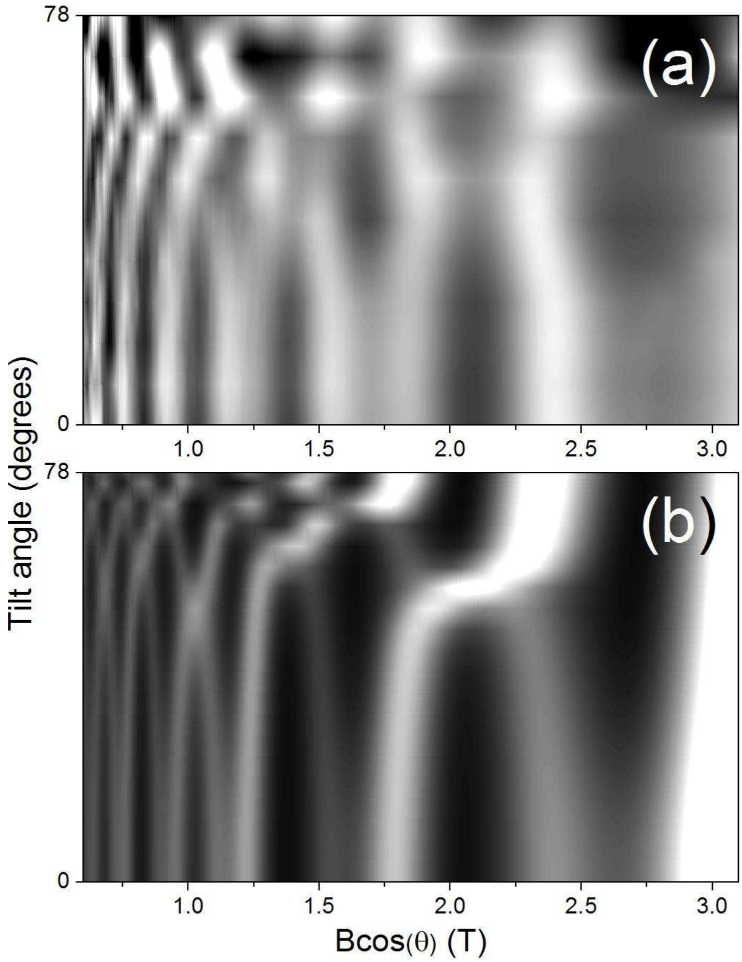

Fig. 2. a) Experimental and b) calculated gray scale maps for the longitudinal magneto resistance R_{xx} as a function of the perpendicular component of the magnetic field $B_\perp = B\cos(\theta)$ and the tilt angle θ, in the range of $B_\perp$ from 0.6 T to 3.1 T.

with reported value of 1.9. The functional form of the dependency of the VS energy (Fig. 2 and Eq. 8) is very different from a simple linear dependency in the overall considered range of values for the perpendicular component of magnetic field ($0.6\ \mathrm{T} \leq B_\perp \leq 6.0\ \mathrm{T}$). This contrasts to the results of Ref. 1, despite the fact that qualitatively we also verified a tendency to $\Delta E_{VS,\nu}$ be lowly affected by the field as we go to $B_\perp \to 0$, and a linear behavior in the approximate range of magnetic fields $\sim 1.4\ \mathrm{T} \leq B_\perp \leq 2.2\ \mathrm{T}$. Therefore this linearity and the convergence to a constant value as $B_\perp \to 0$ T are qualitatively equal to the results of Ref. 1. But the new nonlinear behavior at higher fields ($2\ \mathrm{T} < B_\perp$) should not to be considered as a bad result: in fact, the positions of the squares in Fig. 2 follow – particularly at higher fields – a behavior more closely to the expected square root dependency

$\Delta E_{VS} \propto B_{\perp}^{1/2}$ that comes from a many-body interaction model, where the significant parameter is the inverse of the magnetic length.

References

1. Y. P. Shkolnikov, E. P. De Poortere, E. Tutuc, and M. Shayegan, *Phys. Rev. Lett.* **89**, 226805 (2002).
2. T. Ando, A. B. Fowler and F. Stern, Rev. Mod. Phys. 54 (2) (1982); T. Ando, *Surf. Sci.* **98**, 327 (1980).
3. F. F. Fang and P. J. Stiles, *Phys. Rev.* **174**, 823 (1968).
4. G. Gobsch, D. Schulze, and G. Paasch, *Phys. Rev. B* **38**, 10943 (1988).
5. C. A. Duarte, G. M. Gusev, A. A. Quivy, T. E. Lamas, A. K. Bakarov and J. C. Portal, *Phys. Rev. B* **76**, 75346 (2007).
6. D. R. Leadley, R. J. Nicholas, J. J. Harris, and C. T. Foxon, *Phys. Rev. B* **58**, 13036 (1998).
7. M. M. Fogler and B. I. Shklovskii, *Phys. Rev. B* **52**, 17366 (1995).
8. S. J. Papadakis, E. P. De Poortere, and M. Shayegan, *Phys. Rev. B* **59**, 12743 (1999).