A Letters Journal Exploring the Frontiers of Physics

November 2009

EPL, **88** (2009) 47007
doi: 10.1209/0295-5075/88/47007

www.epljournal.org

Magnetic-field asymmetry of nonlinear transport in a small ring

G. M. Gusev[1], Z. D. Kvon[1(a)], E. B. Olshanetsky[2] and A. Y. Plotnikov[2]

[1] *Instituto de Física da Universidade de São Paulo - CP 66318 CEP 05315-970, São Paulo, SP, Brazil*
[2] *Institute of Semiconductor Physics - Novosibirsk 630090, Russia*

received 23 August 2009; accepted in final form 5 November 2009
published online 2 December 2009

PACS 73.23.-b – Electronic transport in mesoscopic systems
PACS 73.63.-b – Electronic transport in nanoscale materials and structures

Abstract – We have investigated the magnetic-field asymmetry of the conductance in the nonlinear regime in a small Aharonov-Bohm ring. We have found that the odd-in B and linear in V (the DC bias) correlation function of the differential conductance exhibits periodical oscillations with the Aharonov-Bohm flux. We have deduced the electron interaction constant and analyzed the phase rigidity of the Aharonov-Bohm oscillations in the nonlinear regime.

Introduction. – The interest in the quantum magnetotransport in mesoscopic systems has been recently renewed by the suggestion that the nonlinear transport in semiconductor quantum dots and rings [1–3] does not obey the Casimir-Onsager symmetry rules in magnetic field. The linear transport of the mesoscopic sample in a two-terminal configuration is always an even function of the magnetic field, which follows from the time reversal symmetry and the sign of the entropy production rate [4]. However, it has been shown that in the nonlinear regime mesoscopic fluctuations of the local current densities lead to violations of the Onsager relation [5–7]. Beyond the linear regime it is important to study the quadratic voltage response and verify its symmetry rules in magnetic field. One can introduce $g_{s,as} = [g(B) \pm g(-B)]/2$, where $g = \mathrm{d}I/\mathrm{d}V$ is the differential conductance. The amplitude of the conductance fluctuations has been calculated for quantum dots and Aharonov-Bohm rings using the Landauer formula approach [5,8] and the diagram technique, developed in [6,7]. For weak interactions both these approaches predict the following rms amplitude: $\langle\Delta g_{as}\rangle = \langle(g_{as}(B) - \langle g_{as}(B)\rangle)^2\rangle^{1/2}$ of the asymmetric part of the conductance fluctuations:

$$\langle\Delta g_{as}\rangle \approx \alpha\left(\frac{4e^4}{h^2 g}\right)\left(\frac{eV}{E_T}\right) f(x), \tag{1}$$

where $\langle g_{as}(B)\rangle$ is the monotonic component of $g_{as}(B)$ and the outer triangular brackets denote averaging over the magnetic field B, $x = \frac{BL^2}{\Phi_0}$, $\Phi_0 = h/e$ is the magnetic flux, $f(x)$ is equal to x for $0 < x < 1$ and 1 for $x \gg 1$, $E_T = h/\tau_D$ is the Thouless energy, $\tau_D = L^2/D$ is the time it takes for an electron to diffuse across a sample of size L, α is the interaction constant. Note that for $x \gg 1$ and $g \sim 2e^2/h$ we obtain $\frac{\langle\Delta g_{as}\rangle}{\langle\Delta g_s\rangle} \approx \alpha(\frac{eV}{E_T})$, where $\langle\Delta g_s\rangle \sim \langle\Delta g\rangle \sim \frac{2e^2}{h}$ is the rms amplitude of the symmetric component of the conductance fluctuations. Therefore, from the measurements of the asymmetric component of the differential conductance it is possible to derive the value of the interaction constant α. Indeed in the absence of the electron interaction the asymmetric component of the nonlinear conductance fluctuations vanishes, and the Onsager relation is recovered.

The asymmetric component of the nonlinear conductance and the interaction constant have been measured in the chaotic quantum dot [1], carbon nanotubes [9] and the mesoscopic Aharonov-Bohm ring [3]. Despite the unambiguous demonstration of the magnetic-field asymmetry of the nonlinear conductance in such systems several questions still remain. In the present work we perform systematic measurements of the differential conductance in the regime $\langle\Delta g\rangle \sim \frac{2e^2}{h}$ as a function of the applied bias and in the wide range of the magnetic field. Since our ring has a smaller diameter than that in ref. [3], we are able to measure directly the conductance (and not the DC rectification). In addition we estimate the Thouless energy from the gate voltage Aharono-Bohm (AB) oscillations dependence and analyze the phase rigidity in the nonlinear regime. We derive the interaction constant from the comparison with the theory and obtain the value of $\alpha \approx 0.65 \pm 0.1$.

(a) Permanent address: Institute of Semiconductor Physics - Novosibirsk 630090, Russia.

Experimental results and discussions. – The ring interferometer investigated in this paper has been fabricated on the basis of the AlGaAs/GaAs heterostructures with a shallow (25 nm below the sample surface) layer of a two-dimensional electron gas (2DEG) confined at the heterointerface. The electrons are supplied by a Si-doped plane located at 7.5 nm from the heterointerface. The electron mobility in the initial heterostructure was $\mu = 10^5\,\mathrm{cm^2/Vs}$ at $T = 4.2\,\mathrm{K}$ and the density $n_s = 5\times10^{11}\,\mathrm{cm^{-2}}$, which corresponds to the electron mean free path $l = 1.2\,\mu\mathrm{m}$. The Aharonov-Bohm ring was fabricated in the center of the Hall bar structure by means of a high-resolution electron beam lithography followed by a fast plasma etching, as described in [10]. The ring was connected to the regions in the Hall bar with a two-dimensional electron gas by two point-contacts, and the measurements performed were effectively two terminal. The structure was entirely covered by a Au/Ti gate. The longitudinal resistance was measured using an AC current of 10^{-8} A at a frequency of 6.1 Hz in a four-probe Hall bar setup in the linear regime. Direct electric current I_{DC} was applied simultaneously with AC excitation through the same current leads. Measurements have been performed in magnetic fields up to 1.5 T and in the temperature range 1.5–4.2 K. The resistance, or strictly speaking, the differential resistance r_{xx} has been measured as a function of the gate voltage V_g and perpendicular magnetic field B for different values of the DC bias. In the linear regime at $I_{DC} = 0$ the resistance r_{xx} exhibits nonmonotonic dependence on V_g demonstrating a plateaux-like behavior at $r_{xx} = \frac{h}{2e^2 n}$, where n is an integer number, due to the quantization of the point contact resistance. We found that the Aharonov-Bohm oscillations amplitude has a maximum at the gate voltage $V_g = -25\,\mathrm{mV}$, when the total resistance is close to the value of $r_{xx} \approx h/2e^2$. While similar results were obtained for different values of V_g and after several thermocyclings in this article we will be discussing the measurements performed at $V_g = -25\,\mathrm{mV}$.

The results of the magnetoresistance measurements are shown in fig. 1 for different values of the DC current. All traces show the Aharonov-Bohm oscillations with average period close to 0.2 T, which corresponds to the ring diameter 160 nm. Note that this diameter is 2 times smaller than the diameter of the ring used in ref. [2] and 9 times smaller than that in ref. [3]. The amplitude of the AB conductance oscillations is still much less than $2e^2/h$ and approach 20% of the total conductance (fig. 1(b)). We attribute this lack of total modulation to the asymmetry of the interferometer, which strongly suppress the AB oscillations amplitude [11] even in a small ring with diameter $d \ll L_{\varphi,T}$, where L_φ is the phase coherence length, and L_T is the thermal length (due to the averaging over with temperature) [12–15]. As expected for two-terminal measurements, the linear resistance is found to be symmetric (fig. 1a, b) in magnetic field. This symmetry is broken, when a finite DC current is applied, and the asymmetric component of the conductance oscillations increases with I_{DC}. It is worth noting that the average amplitude of the AB oscillations decreases with DC current, which we attribute to the heating effect. Surprisingly, we find that the AB amplitude decreases more rapidly for the positive side of the magnetic field. We performed the calculation of the rms amplitude of the symmetric and asymmetric components of the conductance fluctuations $\langle\Delta g_{as,s}\rangle = \langle(g_{as,s}(B) - \langle g_{as,s}(B)\rangle)^2\rangle^{1/2}$. The ratio $\langle\Delta g_{as}\rangle/\langle\Delta g_s\rangle$ is shown in fig. 2.

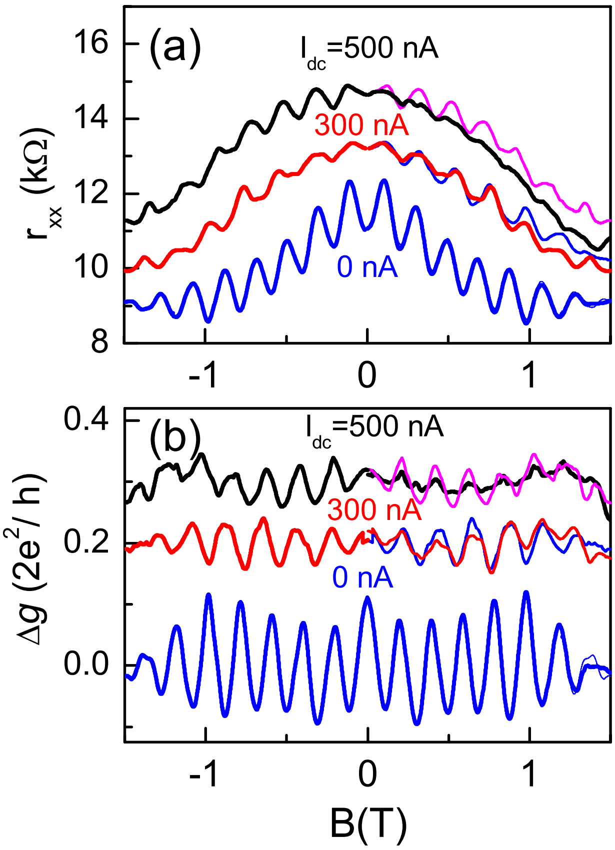

Fig. 1: (Color online) (a) Differential resistance of the ring as a function of the magnetic field for different DC currents at $T = 1.5\,\mathrm{K}$. (b) Differential conductance of the ring as a function of the magnetic field for different DC currents at $T = 1.5\,\mathrm{K}$.

One can see that it is proportional to the applied bias for small values of V_{DC} and saturates at $V_{DC} \approx 3.2\,\mathrm{mV}$ where $\langle\Delta g_{as}\rangle/\langle\Delta g_s\rangle \sim 1$. Generally speaking, such behaviour is consistent with eq. (1) confirming that the linear voltage conductance expansion holds for the DC bias range used. Note that eq. (1) predicts for the rms amplitudes of the conductance oscillations that $\langle\Delta g_{as}\rangle \gg \langle\Delta g_s\rangle \gg 2e^2/h$ in the regime $eV \gg E_T$, which hardly agrees with the mesoscopic theory. It is therefore very likely that Δg_{as} saturates at $V \approx E_T$. In general, the energy $eV_c \sim E_T$ is supposed to be the energy scale for the crossover from linear to nonlinear transport, since even without interaction nonlinear conductance exists [16]. Theory predicts that at $eV_c \approx E_T$ we have $\frac{\langle\Delta g_{as}\rangle}{\langle\Delta g_s\rangle} \approx \alpha$, and therefore we obtain $\alpha = 0.8 \pm 0.2$. Equation (1) has been derived in the limit of weak

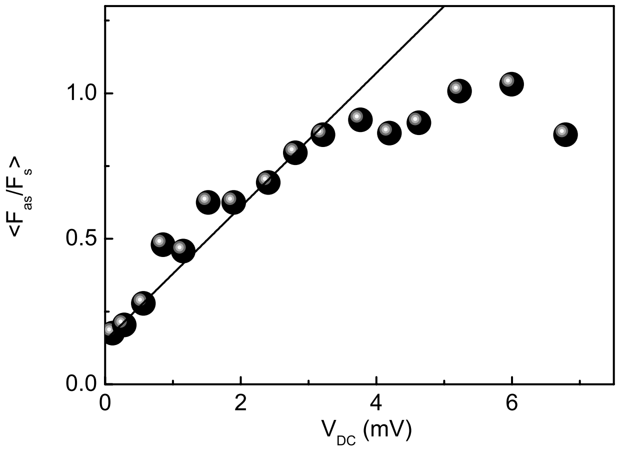

Fig. 2: Normalized rms amplitude $\langle \Delta g_{as}(B)\rangle/\langle \Delta g_s\rangle$ for different DC currents at $T=1.5\,\text{K}$.

interaction ($\alpha \ll 1$), which is obviously not quite applicable to our situation [6,7]. A more general theory that accounts for nonlinear effects at arbitrary magnetic field and interaction strength has been developed in [5,8,17] within the Landauer formula approach. It has been shown that the ratio of charging energy, determined by the capacitance C of the quantum dot (ring) $E_c = e^2/2C$ to the mean level spacing $\Delta = h v_F/\pi L$, where v_F is Fermi velocity, characterizes the interaction strength. For example, that theory predicts $\alpha = 1/(1+\Delta/E_c)$, which in the limit $\Delta \gg E_c$ gives the value of $\alpha \approx \Delta/E_c \ll 1$ [5]. In the limit of strong interactions the average amplitude of the asymmetric contribution to the conductance fluctuations is given by [17]

$$\langle \Delta g_{as}\rangle \approx \left(\frac{2e^2}{h}\right)\left(\frac{eV}{E_T}\right)\frac{1}{\sqrt{1+2(gh/2e^2)^2(1/\alpha-1)^2}}. \quad (2)$$

In our case for $g \sim \Delta g_s \sim 2e^2/h$ and at $eV_c \approx E_T$ we obtain

$$\frac{\langle \Delta g_{as}\rangle}{\langle \Delta g_s\rangle} \approx \frac{1}{\sqrt{1+2(1/\alpha-1)^2}}. \quad (3)$$

From the comparison of the experimental results (fig. 2) with eq. (3) we obtain $\alpha \approx 0.65$. This is not surprising since the estimation of the mean level spacing in our ring gives $\Delta \approx 2.2\,\text{meV}$. On the other hand, the capacitance energy may be determined from the period of the Coulomb blockade oscillations in the ring [18], which gives $E_c \approx e^2/\varepsilon L \approx 3.3\,\text{meV}$ [18,19]. Finally from the ratio $\Delta/E_c = 0.67$ we obtain $\alpha \approx 0.65 \pm 0.1$, which is close to the experimental value. It is worth noting that a similar interaction constant has been obtained from the measurements of the rectification current in a ring with the diameter 9 times larger than in our sample [3]. However, it seems that neither the capacitance energy nor the energy level spacing have been measured in this work.

In order to obtain the information about energy level spacing in our ring we have measured AB oscillations for different gate voltages as shown in fig. 3. One can see that as the gate voltage is varied, the phase of the AB oscillations φ switches between 0 and 2π. It is worth noting that the AB oscillations with $\varphi = 0$ evolve into the AB oscillations with $\varphi = \pi$ via an intermediate state of $h/2e$ oscillations [20]. The change of the AB phase by 2π corresponds to the shift of the Fermi level by the energy level separation in the ring at zero magnetic field $\Delta = h v_F/\pi d$. Comparing our results with exact spectrum of the isolated ring in the magnetic field [21] we obtain $\Delta \approx 2.6\,\text{meV}$, very close to the value used in our estimations. Note that the Thouless energy in the ring is determined by formula $E_T = \hbar \min\{l v_F/L^2, v_F/L\}$, and in the ballistic regime it coincides with the level spacing. From fig. 2 we find that the saturation of the amplitude of the nonlinear conductance oscillations occurs at $eV_{DC} \approx 3.2\,\text{meV} \sim E_T$, which agrees well with the value Δ, obtained from the gate voltage dependence (fig. 3). Therefore one can suppose that both the energy scale for the crossover from a linear to a nonlinear regime and the level spacing in the ring are relevant energy scales in the problem.

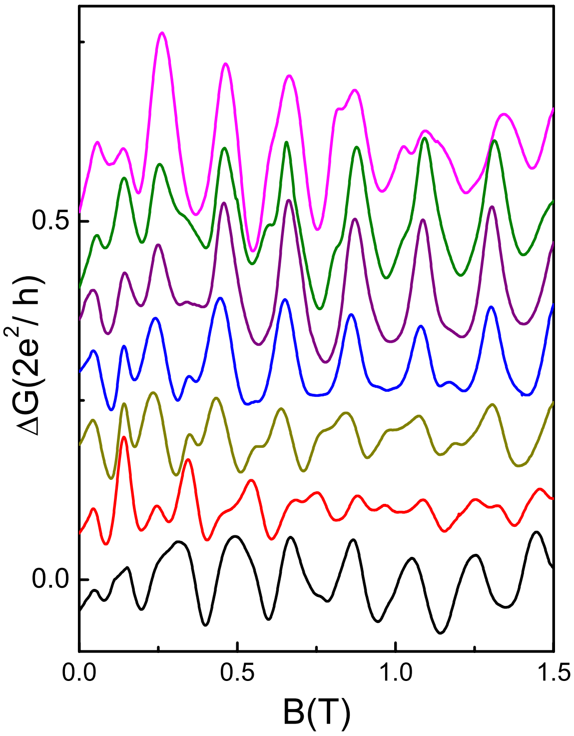

Fig. 3: (Color online) Conductance of the ring as a function of the magnetic field for different V_g at $T=1.5\,\text{K}$ (from bottom to top V_g (mV): 25, 30, 35, 40, 45, 50, 55).

In the last section of the article we will discuss the fluctuation of the phase of the AB oscillations in a nonlinear regime. Since the Coulomb interaction results in the asymmetry of the nonlinear conductance oscillations, the phase of these oscillations is no longer pinned at 0 or π. Figure 4 shows AB oscillations similar to those in fig. 1, but taken with a smaller step in DC current in order to emphasize the nontrivial behaviour of the

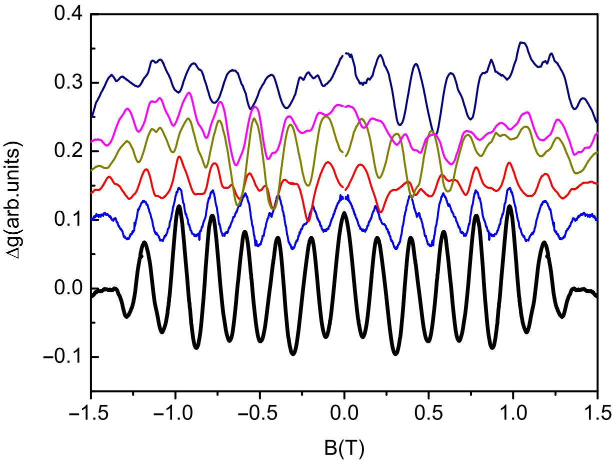

Fig. 4: (Color online) Conductance of the ring as a function of the magnetic field for different DC currents at $T = 1.5$ K (from bottom to top I_{DC} (nA): 0, 50, 75, 160, 200, 400).

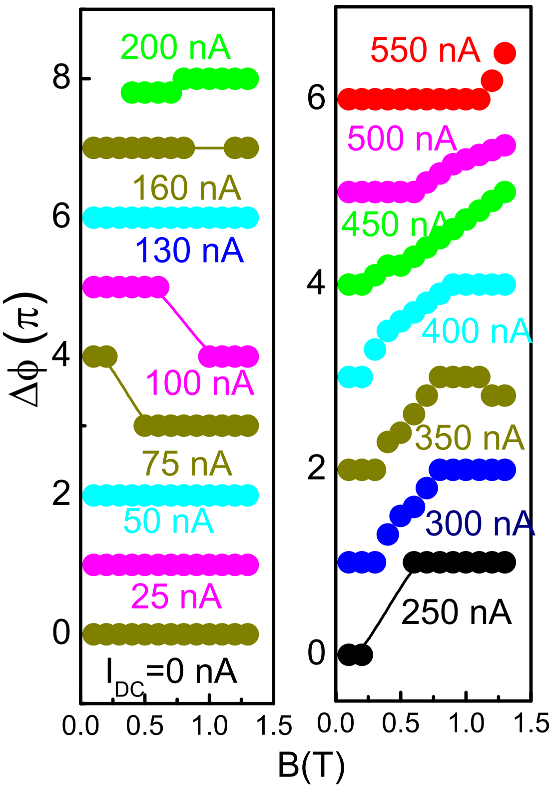

Fig. 5: (Color online) The phase of the AB oscillation as a function of the magnetic field for different values of the DC current. Each data set is π-phase shifted vertically for clarity.

phase in a nonlinear regime. One can see that at low DC currents the AB oscillations behave as $\Delta g(B) = g(B) - \langle g(B)\rangle \sim \frac{2e^2}{h}\cos(B/B_0 + \varphi)$, where B_0 is the period of the oscillations, while the phase $\varphi = 0$ and does not depend on the magnetic field. Surprisingly, in a nonlinear regime, when the B-symmetry of the oscillations is broken, the phase becomes B-dependent. One can see that at zero magnetic field the phase switches from 0 to π with the DC current increasing, it remains practically constant up to a certain magnetic field B_c and then switches again from π to 0 at $B > B_c$. At higher DC currents several switches of the phase between values 0 and π are observed near zero magnetic field, and, in addition, the critical magnetic field B_c becomes higher. The results for the phase shift versus DC current are summarized in fig. 5. Note, that this behaviour is markedly different from the variation of the phase with V_g (fig. 3), where the phase does not depend on magnetic field [20]. The transition of the phase from 0 to π and vice versa in a nonlinear regime occurs in a narrow interval of magnetic field at a low DC bias, becoming broader for higher values of the DC current, where we have also observed a continuous phase shift (see fig. 5). The destruction of the phase rigidity of nonlinear conductance oscillations has been observed in a previous study [2]. The originality of our result, however, is related to the fact that we find variation of the phase for *linear* conductance fluctuations in a nonlinear regime. For example the B-dependent phase at $I_{DC} = 75$ nA cannot be explained by an asymmetric nonlinear contribution, since $\langle \Delta g_{as}\rangle \ll \langle \Delta g\rangle$.

Itis worth noting that a large body of theoretical work has been devoted to the study of the phase behaviour in the Aharonov-Bohm interferometer with a quantum dot embedded in one of the arms (for a review see [22]). Indeed, the theoretical model [23] predicted that in a nonlinear regime the phase rigidity may be broken, and phase may change continuously. However, only recently phase variation tuned by a bias voltage has been experimentally studied in an AB interferometer with two tunnel barriers in the arms in the quantum Hall effect regime [24], and in a two-dot ring [25] near zero magnetic field. Our ring geometry is very different from both the two-barrier and the two-dot AB ring geometries and, therefore, the explanation of the observed effect as resulting from electrostatic AB oscillations [24] or interactions in the quantum dot [23] is hardly possible. The observed variation of the phase of the AB oscillations as a function of DC bias and magnetic field in nonlinear regime requires further theoretical study.

Conclusion. – We have systematically analyzed AB oscillations in the resistance of a 160 nm GaAs ring in a single-mode regime and in the presence of a DC bias. As expected for two-terminal nonlinear transport the magnetic-field symmetry is broken. Due to a small diameter of the ring we have been able to observe the violation of the Onzager relation when $g \sim 2e^2/h \sim \Delta g \sim \Delta g_{as}$. We have independently measured the Thouless energy, the energy level spacing and the capacitance energy in our ring and determined the interaction strength constant α, which is responsible for the asymmetric nonlinear conductance in a mesoscopic system. This constant is consistent with the value derived from the comparison of the B-asymmetric AB oscillations with the theory. We have analyzed the phase rigidity of the AB oscillations in the nonlinear regime and found that the phase switches by

$\Delta\varphi = \pi$ with magnetic field increasing. We cannot explain such phase variation by the asymmetry of the nonlinear conductance oscillations.

* * *

Support of this work by FAPESP, CNPq (Brazilian agencies), RFBI (grant No. 08-02-01007), RFBR (02-08-01007) and programs of RAS Physics of Nanostructures and Nanoelectronics and Quantum Physics of Condense Matter is acknowledged.

REFERENCES

[1] Zumbuhl D. M., Marcus C. M., Hanson M. P. and Gossard A. C., *Phys. Rev. Lett.*, **96** (2006) 206802.
[2] Leturcq R., Sanchez D., Gotz G., Ihn T., Ensslin K., Driscoll D. C. and Gossard A. C., *Phys. Rev. Lett.*, **96** (2006) 126801.
[3] Angers L., Zakka-Bajjani E., Deblock R., Gueron S., Bouchiat H., Cavanna A., Gennser U. and Polianski M., *Phys. Rev. B*, **75** (2007) 115309.
[4] Landau L. D. and Lifshitz E. M., *Statistical Physics* (Butterworths, Heinemann, London) 1989.
[5] Polianski M. L. and Buttiker M., *Phys. Rev. Lett.*, **96** (2006) 156804.
[6] Spivak B. and Zyuzin A., *Phys. Rev. Lett.*, **93** (2004) 226801.
[7] Deyo E., Spivak B. and Zyuzin A., *Phys. Rev.*, **74** (2006) 104205.
[8] Polianski M. L. and Buttiker M., *Phys. Rev. B*, **76** (2007) 205308.
[9] Wei J., Shimogawa M., Wang Z., Radu I., Dormaier R. and Cobden D. H., *Phys. Rev. Lett.*, **95** (2005) 256601.
[10] Kvon Z. D., Kozlov D. A., Olshanetsky E. B., Plotnikov A. Y., Latyshev A. V. and Portal J. C., *Solid State Commun.*, **147** (2008) 230.
[11] Tkachenko V. A., Kvon Z. D. and Sheglov D. V., *JETP Lett.*, **79** (2004) 136.
[12] Webb R. A., Washburn S., Umbach C. P. and Laibowitz R. B., *Phys. Rev. Lett.*, **54** (1985) 2696.
[13] Lee P. A., Stone A. D. and Fukuyama H., *Phys. Rev. B*, **35** (1987) 1039.
[14] Casse M., Kvon Z. D., Gusev G. M., Olshanetskii E. B., Litvin L. V., Plotnikov A. V., Maude D. K. and Portal J. C., *Phys. Rev. B*, **62** (2000) 2624.
[15] Hansen A. E., Kristensen A., Pedersen S., Sorensen C. B. and Lindelof P. E., *Phys. Rev. B*, **64** (2001) 045327.
[16] Altshuler B. L. and Khmelnitskii D. E., *JETP Lett.*, **42** (1985) 359.
[17] Polianski M. L. and Buttiker M., *Physica E*, **40** (2007) 67.
[18] Tkachenko V. A., Kvon Z. D. and Sheglov D. V., *JETP Lett.*, **79** (2004) 136.
[19] Mayorov A. S., Kvon Z. D., Savchenko A. K., Sheglov D. V. and Latyshev A. V., *Physica E*, **40** (2008) 1121.
[20] Olshanetskii E. B., Casse M., Kvon Z. D., Gusev G. M., Litvin L. V., Plotnikov A. V., Maude D. K. and Portal J. C., *Physica E*, **6** (2000) 322.
[21] Tan W. C. and Inkson J. C., *Phys. Rev. B*, **53** (1996) 6947.
[22] Kouwevhoven L. P. *et al.*, in *Mesoscopic Electron Transport*, edited by Kouwenhoven L. P., Schon G. and Sohn L. L., *NATO ASI Ser. E*, Vol. **345** (Kluwer, Dordrecht) 1997, pp. 105–214.
[23] Bruder C., Fazio R. and Schoeller H., *Phys. Rev. Lett.*, **76** (1996) 114.
[24] van der Wiel W. G, Nazarov Yu. V, De Franceschi S., Fujisawa T., Elzerman J. M., Huizeling E. W. G. M., Tarucha S. and Kouwenhoven L. P., *Phys. Rev. B*, **67** (2003) 033307.
[25] Sigrist M., Ihn Thomas, Ensslin K., Reinwald M. and Wegscheider W., *Phys. Rev. Lett.*, **98** (2007) 036805.