Evolution of the two-dimensional towards three-dimensional Landau states in wide parabolic quantum well

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Abstract

Shubnikov-de Haas oscillations are measured in wide parabolic quantum wells with 2 subbands occupied in tilted magnetic field. Experiments reveal anticrossing of the Landau level (LL) belonging to lowest subband and last LL belonging to the second subband. Such anticrossing occurs due to decrease of the energy of the LL with tilt angle $Q$. This observation is supported by the measurements of the $Q$-dependence of the activation energy in the quantum Hall effect regime.

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1. Introduction

Wide parabolic quantum well (PQW) is the natural extension of the two-dimensional (2D) system towards fully three dimensionally (3D) engineered quantum structures [1]. In strong magnetic field subband spectrum of the parabolic well is transformed into series of Landau levels. The LL energy is strongly depend on the experimental geometry and may be governed by the magnetic field tilting experiments, since energy spectrum in wide quantum well depends on the angle between magnetic field and normal to the quantum well plane [2]. When magnetic field becomes exactly parallel to the well plane, 2D Landau states transforms to the 3D states. Evolution of the energy levels in the titled field in PQW has been studied in paper [3] for 1000 Å parabolic well. The states in tilted field, so called oblique states, transform into plane waves along the field in the bulk Landau states, since their localization length $W_x$ grows as $W_x = \tan \Theta$, when field is tilted in the $x$-direction. The oblique states have characteristics of the 1D states and may display some interesting properties due to the strong Coulomb interaction. In present paper we performed detailed measurements of the magnetoresistance in the titled magnetic field for parabolic well with different widths. Anticrossing of the LL belonging to the different subbands demonstrates the evolution of the 2D towards 3D Landau states.

In a magnetic field directed along $z$-axis, perpendicular to the well plane, each subband represents a stair-case of LL associated with the subband energy $E^{\text{sub}}_i$ ($i$ is the subband index). The energy spectrum of the electron states is given by

$$E_n = E^{\text{sub}}_i + \hbar \omega_i (n + 1/2)$$

(1)

The energy of the electrons in a PQW with the potential $V = (az)^2$ in the presence of an in-plane magnetic field oriented along $x$-axis is given by

$$E_n = \frac{\hbar^2}{2m} (k_x^2 + \gamma k_y^2 + \hbar \omega_i (n + 1/2))$$

(2)

where $\omega^2 = \omega_0^2 + \omega_i^2$; $\omega_0^2 = a^2 \sqrt{2m}$; $\gamma = \omega_0^2 / \omega_i^2$, and $m$ is effective mass.

For wide parabolic wells in strong magnetic field $\omega_0 \gg \omega_i$, we have

$$E_n = \frac{\hbar^2 k_x^2}{2m} + \hbar \omega_i (n + 1/2)$$

(3)

which is the energy of the 3D Landau subbands, where $n$ is the Landau level number.

From the comparison of the Eqs. (1) and (3) we may see that 2D energy spectrum is gradually transformed to 3D energy spectrum in the strong magnetic field $B$, when $B$ is tilted away from normal to the quantum well plane.

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2. Experiment and discussion

The samples are the GaAs/Al\(_{x}\)Ga\(_{1-x}\)As PQWs grown on undoped (100) GaAs substrate by molecular-beam epitaxy. On the top of the substrate there is 10,000 Å GaAs buffer layer with 20 periods of AlAs(5 ML) GaAs (10 ML) superlattice, followed by 5000 Å Al\(_{x}\)Ga\(_{1-x}\)As with \(x\) varying from 0.0 to 0.30. The structure consists of the 1000, 2000 and 3000 Å wide Al\(_{x}\)Ga\(_{1-x}\)As well in which \(x\) was quadratically varied between \(x = 0\), at the centre of the well, and \(x = 0.27\), at the edges of the well. On each side, the well is bounded by Si-doped (\(5 \times 10^{11}\) cm\(^{-2}\)) Al\(_{0.3}\)Ga\(_{0.7}\)As layers, grown next to spacer layers. The thicknesses of the undoped Al\(_{0.3}\)Ga\(_{0.7}\)As space layers are 100 Å. A 100 Å GaAs cap layer was grown as final layer of the structure.

After growth, are photolithographically defined Hall bar with dimensions 200 × 500 μm\(^2\). Four-terminal resistance and Hall measurements were made down to 50 mK in magnetic field up to 15 T. The measurements were performed with an ac current not exceeding 10\(^{-8}\) A. Resistance was measured for different angles \(\theta\) between the field and normal to the substrate plane in magnetic field using an in situ rotation of the sample.

The mobility of the electron gas in the well is \(\mu_H = 171 \times 10^3\) cm\(^2\)/V·s (PQW-1000-Å), \(\mu_H = 151 \times 10^3\) cm\(^2\)/V·s (PQW-2000-Å) and \(\mu_H = 118 \times 10^3\) cm\(^2\)/V·s (PQW-3000-Å); and the electron concentration allowed to obtain two occupied subbands.

Fig. 1 shows the field dependence of the Shubnikov-de Haas oscillations for different angles \(\theta\). (In this paper, we only will show the results for PQW-2000-Å.) All samples reveal the resonance Landau level coupling, i.e. anticrossing of the occupied Landau level belonging to first and second subbands. Fig. 1 show the grey scale presentation of the anticrossing of the levels, identified by B1, B2 and B3 (first subband) and levels A1 and A2 belonging to the second subband.

Fig. 2 shows anticrossing of the LL in the low perpendicular magnetic field and large tilt angles. Since the energy gaps between levels becomes very small, it is not possible to detect anticrossing for LL with higher quantum number. It is expected the 3D limit, shown in Fig. 3 may be reached when \(\theta \to 90^\circ\) and the distance...
between 2D Landau subbands $\Delta E_{ij} \rightarrow 0$. In a real system the energy levels have a finite width due to the disorder $\Gamma$, therefore, the electron system has a 3D energy spectrum when the electron subbands overlap, $\Gamma \sim \Delta E_{ij}$, which probably occurs in the interval $\Theta \sim 85-90^\circ$. It is expected that all 2D LL belonging to the first subband are abruptly collapsed into 3D Landau state, as can be seen in Fig. 3, when $\Theta \rightarrow 90^\circ$. However, in more realistic scenario upper 2D Landau levels start to collapse to bulk LL before lowest one. It may lead to the coexistence of the 3D and 2D LL at $\Theta$, 80–85°, which we indeed observed in experiments (not shown). We attribute thus effect to the magnetic field dependence of the single particle relaxation time, which is responsible for the broadening of the levels. It is worth noting, that such coexistence is different for the effects studied in paper [4], when 3D LL were observed in perpendicular magnetic field in wide PQW. In wide well 3D limit has been reached because the energy space between 2D subbands is comparable with level broadening in zero magnetic field.

In strong magnetic field the behaviour of the transverse $R_{xx}$ and Hall resistance $R_{xy}$ of the parabolic well resembles the behaviour of the transport coefficients in the conventional narrow quantum well or GaAs/AlGaAs heterostructures containing 2D electron gas (2DEG): $R_{xy}$ exhibits very well-developed quantum Hall plateaux at integer filling factors $\nu$, and magnetoresistance $R_{xx}$ approaches zero at Hall steps. However, as can be seen in Fig. 3, the energy distance between Landau levels in strong magnetic field is determined by the energy levels separation $\Delta E_{ij}$ in zero field. From the measurements of the activation energy of the resistance in plateaux regime, we can deduce $\Delta E_{ij}$. Taking into account spin splitting, which is responsible for the minima at odd filling factor, we have measured the activation energy in tilted magnetic field for filling factor $\nu = 2$, 4 and 6. Temperatures in the range 1.4–4.2 K were measured taking care to ensure that thermal equilibrium was established before measurements were made. The data for sample PQW-2000-Å $\nu = 2$ were shown in Fig. 4. All plots have substantial straight-line regions. The longitudinal conductivity $\sigma_{xx}$ is given by

$$\sigma_{xx} = \sigma_0 \exp\left(-\frac{\Delta E}{2k_BT}\right)$$  \hspace{1cm} (4)

Then, the energy gaps are calculated from the fit the Arrhenius plots. As we already mentioned above, the energy gap in PQW is given by

$$\Delta E = \frac{\Delta E_{ij}}{\cos(\Theta)} - g^* \mu_B B$$  \hspace{1cm} (5)

Fig. 5 shows the relative gap energy, $\Delta E_{ij} = \Delta E(\Theta)/\Delta E(0^\circ)$ as a function of the tilted angle $\Theta$ for filling factor $\nu = 2$, 4 and 6 and for PQWs with various widths $W = 1000$, 2000 and 3000 Å. We can see that $\Delta E_{ij} \rightarrow 0$, when the angle increases. This observation agrees with scenario shown in Fig. 3.

3. Conclusions

In the present work we have demonstrated that the PQW is promising system for understanding of the energy spectrum evolution in the tilted magnetic field. Decoherence properties of the oblique states can be studied. For example, localization length $W_\ell$ may approach several $\mu$m at $\Theta \sim 85^\circ$. The Coulomb interaction between such 1D states may
be very strong and lead to the new strongly correlated states in the strong magnetic field.

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References