Vertical longitudinal magnetoresistance of semiconductor superlattices

Yu. A. Pusep,^{1,2} G. M. Gusev,¹ A. J. Chiquito,² S. S. Sokolov,^{2,3} A. K. Bakarov,⁴ A. I. Toropov,^{2,4} and J. R. Leite¹

¹Instituto de Fisica da Universidade, São Paulo, 05315-970 São Paulo, São Paulo, Brazil

²Departamento de Fisica, Universidade Federal de São Carlos, Caixa Postale 676, 13565-905 São Carlos, Brazil

³Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 61164 Kharkov, Ukraine

⁴Institute of Semiconductor Physics, 630090 Novosibirsk, Russia

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Vertical longitudinal magnetoresistance (VLMR) caused by the peculiar shape of the Fermi surface of a superlattice has been observed in GaAs/Al_xGa_{1-x}As superlattices. This VLMR occurs when the electrons occupy the open Fermi surface and their motion in the plane of the layers is quantized by a magnetic field. It was shown that there exists a critical magnetic field that cancels the contribution of the electrons occupying the open Fermi surface to the vertical conductivity in the case when the chemical potential exceeds the width of the miniband, thus resulting in the observed VLMR. This effect produces the conditions necessary to observe the quantized Hall effect in the three-dimensional electron system of a superlattice.

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I. INTRODUCTION

During the last decade, there has been considerable interest in studying the integer quantum Hall effect (OHE) in anisotropic three-dimensional electronic systemssemiconductor superlattices (SL's) where the minima of the resistivity ρ_{xx} were found to be accompanied by similar minima of the vertical conductivity σ_{zz} . Thus, a significant role of the vertical conductivity in formation of quantized states in a quasi-three-dimensional electronic system was determined.^{1,2} In later studies two different contributions to the vertical conductivity of a superlattice in a strong magnetic field were distinguished: one due to the bulk and the other caused by the hybridization of the edge states of the superlattice layers.^{3–5} The latter was shown to form the socalled two-dimensional chiral metal, which was observed at low enough temperatures ($T \le 1$ K) when all bulk quantum Hall states are localized and therefore the surface conductivity contributes significantly.

Moreover, the vertical longitudinal magnetoresistance (VLMR) of superlattices (the magnetoresistance that occurs when both electric and magnetic fields are directed along the growth direction of the superlattice) was studied in Refs. 6-11. As a result, specific Shubnikov–de Haas oscillations in the form of resistance resonances were predicted and anisotropy of the vertical magnetotransport in SL's was found: it was shown that the vertical magnetoresistance is much weaker, although still significant, for a magnetic field applied normal to the layers than for one directed parallel to the layers. Thus the observed nonzero VLMR was considered to be the result of nonuniform fluctuations of the widths of the layers, which caused localization of the electrons.^{12,13}

In this paper we present results showing that even in an ideal SL, in appropriate conditions, there exists a strong VLMR originating from the peculiar shape of the Fermi surface of the SL. It is shown that when the Fermi energy lies in a minigap, in the case when the inter-Landau-level spacing is larger than the electron broadening energy (\hbar/τ) , the vertical conductivity tends to zero in a magnetic field applied parallel to the growth direction of the SL. In this case, the three-

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dimensional electron system of a SL behaves like a stack of two-dimensional quantum Hall conductors thus producing conditions necessary for observation of the QHE.

The paper is organized as follows. The theory is considered in Sec. II. The electronic properties of the samples are characterized in Sec. III. The experimental results together with their discussion are given in Sec. IV, and conclusions are outlined in Sec. V.

II. THEORY

As shown in Refs. 14 and 15, at T=0 the vertical conductivity of a SL (along the *z* direction) can be written in the Drude form

$$\sigma_z = \frac{n^* e^2 \tau}{m_z} \tag{1}$$

with the effective electron concentration

$$n^* = \frac{m_{\parallel} W_{SL}}{2 \pi^2 \hbar^2 D_{SL}} \int_0^{k_F} dk_z \left| \frac{v_z(\mathbf{k})}{v_{\max}} \right|^2, \qquad (2)$$

where m_z and m_{\parallel} are the effective masses normal and parallel to the layers, respectively, W_{SL} is the miniband width, D_{SL} is the period of the SL, and v_{max} is the maximum electron velocity. The electron velocity v_z along the superlattice axis, which is responsible for the vertical conductivity of an ideal SL, is determined by the electron spectrum $E(\mathbf{k})$ along all possible directions of the wave vector \mathbf{k} :

$$v_z(\mathbf{k}) = \frac{1}{\hbar} \frac{\partial E(\mathbf{k})}{\partial k_z}.$$
(3)

Thus, a nonvanishing vertical conductivity is expected unless electron dispersion exists. When the Fermi energy exceeds the miniband width, the vertical conductivity is completely determined by the contributions of the electron states that belong to the open Fermi surface of the SL (which has the topology of an undulating cylinder oriented along the SL axis) and it is independent of the electron density while the



FIG. 1. Schematic cross section of the Fermi surfaces in a superlattice (dotted lines). Solid lines show the modification of open Fermi surfaces under a magnetic field in the cases when $\hbar \omega_c < W_{SL}$ (a) and $\hbar \omega_c > W_{SL}$ (b). Thin arrows show the electron interlevel transitions caused by the perturbation potential $U_{int}(\omega,k)$ that contribute to the vertical conductivity. The vertical components of the velocity of electrons occupying the open Fermi surfaces in the absence of the magnetic field are depicted by thick arrows.

Fermi level is located in a minigap (when the Fermi surface remains unchanged). Such behavior of the vertical conductivity was experimentally observed in SL's in Ref. 16. A modification of the Fermi surface of a SL is shown schematically in Fig. 1, where the closed Fermi surfaces are associated with the positions of the Fermi level inside a miniband, while the open Fermi surfaces correspond to the case when the Fermi level enters a minigap.

If now a strong enough magnetic field is applied along the superlattice axis, the in-plane motion of electrons becomes quantized into a series of Landau levels without affecting the miniband dispersion along the superlattice axis; then the energy dispersion becomes

$$E(\mathbf{k}) = E_z(k_z) + (n + \frac{1}{2})\hbar\omega_c, \qquad (4)$$

where $\hbar \omega_c$ denotes the cyclotron energy and n = 0, 1, 2, ...

For a magnetic field *B* such that $\hbar \omega_c > \hbar/\tau$ (where τ is the relaxation time) the vertical motion of an electron has purely one-dimensional character. In this case, for the open Fermi surfaces $v_z(\mathbf{k}) = 0$ and the conductivity of the SL vanishes. Therefore, a magnetic field induced metal-to-insulator transition is predicted to occur for the vertical conductivity when the increasing Fermi energy crosses the top of a miniband at a fixed magnetic field passes through this critical value in a SL with the Fermi level located in a minigap. The critical magnetic field is associated with the relation

$$B_c = \frac{\hbar m}{e \tau}.$$
 (5)

These considerations allow us to conclude that at an appropriate magnetic field $(B>B_c)$ the value of the VLMR could be even larger than that of the vertical transverse mag-

netoresistance (VTMR)—the magnetoresistance associated with magnetic field perpendicular to the vertical current.

We developed a model to account for the observed VLMR. In this model the conductivity is determined by the number of broadened electron states at the Fermi surface that belong to different Landau levels and are mutually coupled by the scattering mechanism. In the following, we consider the electron transitions caused by elastic scattering of electrons by impurities in the presence of the external electric field E_0 . The conductivity can be calculated by means of the matrix density method as proposed in Ref. 17. Following this method, the Hamiltonian can be written in the form

$$\hat{H}_{int} = CU_{int}(\mathbf{r})E_0 \exp(-i\omega t), \qquad (6)$$

where $U_{int}(\mathbf{r}) = Ze^2 \sum_{j=1}^{N_j} [\exp(-\varkappa |\mathbf{r} - \mathbf{r}_j|)/|\mathbf{r} - \mathbf{r}_j|]$ is the potential of the interaction, N_j is the number of impurities, \varkappa is the reciprocal screening length, and *C* is the coefficient of proportionality satisfying the necessary dimensionality.

In a linear approximation the average value of any operator is determined as

$$\langle \hat{\Phi} \rangle = \operatorname{Sp}(\rho_0 \Phi) + C \sum_{\mathbf{k}' \, \alpha', \mathbf{k} \alpha} \frac{[f_0(\varepsilon_{\mathbf{k}' \, \alpha'}) - f_0(\varepsilon_{\mathbf{k} \alpha})]}{\varepsilon_{\mathbf{k}' \, \alpha'} - \varepsilon_{\mathbf{k} \alpha} - i\hbar/\tau} E_0$$

$$\times \exp(-i\omega t),$$
(7)

where the equilibrium value of the density matrix $\rho_0 = \rho(-\infty)$ and $f_0(\varepsilon_{\mathbf{k}\alpha}) = [\exp(\varepsilon_{\mathbf{k}\alpha} - \xi)/kT]^{-1}$ is the Fermi function, which defines the probability of finding an electron in the state $|\mathbf{k}\alpha\rangle$ with energy $\varepsilon_{\mathbf{k}\alpha}$, ξ is the chemical potential, and τ is the relaxation time. Thus, the generalized susceptibility can be calculated according to the formula¹⁷

$$\chi = \sum_{\mathbf{k}'\,\alpha',\mathbf{k}\alpha} \frac{\left[f_0(\varepsilon_{\mathbf{k}'\,\alpha'}) - f_0(\varepsilon_{\mathbf{k}\alpha})\right]}{\varepsilon_{\mathbf{k}'\,\alpha'} - \varepsilon_{\mathbf{k}\alpha} - i\hbar/\tau} (\hat{H}_{int})_{\mathbf{k}'\,\alpha',\mathbf{k}\alpha} \hat{\Phi}_{\mathbf{k}'\,\alpha',\mathbf{k}\alpha}.$$
 (8)

The conductivity can be obtained by assuming that $\hat{\Phi}$ is the operator of the current density $\hat{j}_z = (e/mV)\hat{p}_z$ (where V is the crystal volume):

$$\sigma = \operatorname{Re}\left\{ C \sum_{\mathbf{k}'\alpha',\mathbf{k}\alpha} \frac{[f_0(\varepsilon_{\mathbf{k}'\alpha'}) - f_0(\varepsilon_{\mathbf{k}\alpha})]}{\varepsilon_{\mathbf{k}'\alpha'} - \varepsilon_{\mathbf{k}\alpha} - i\hbar/\tau} \times \langle \mathbf{k}'\alpha' | \hat{U}_{int} | \mathbf{k}\alpha \rangle \langle \mathbf{k}\alpha | \hat{j}_z | \mathbf{k}'\alpha' \rangle \right\}.$$
(9)

In the calculations of the matrix elements of the interaction with impurities we can set $|\mathbf{k}\alpha\rangle = (1/\sqrt{V})\exp(i\mathbf{k}\cdot\mathbf{r})$ because the rapidly oscillating part of the Bloch function does not contribute significantly to the matrix element calculated over large distances $\varkappa^{-1} \ge a$, where *a* is the lattice constant. The result is

$$\langle \mathbf{k}' \, \alpha' \, | \, \hat{U}_{int} | \, \mathbf{k} \alpha \rangle = \frac{4 \, \pi Z e^2}{V[\, \varkappa^2 + |\mathbf{k}' - \mathbf{k}|^2]} \sum_{j=1}^{N_j} e^{-i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}_j}.$$
 (10)

However, the full Bloch function

$$|\mathbf{k}\alpha\rangle = (1/\sqrt{V})\exp(i\mathbf{k}\cdot\mathbf{r})u_{\mathbf{k}\alpha}(\mathbf{r})$$

should be used when calculating the matrix element of the current density.¹⁷ Then, taking into account that $\hat{j}_z = (e/mV)\hat{p}_z = -(e/mV)i\hbar\partial/\partial z$ we obtain

$$\langle \mathbf{k}\alpha | \hat{j}_z | \mathbf{k}' \, \alpha' \rangle = \frac{1}{N} \, \delta_{\mathbf{k}'\mathbf{k}} a_{\alpha\alpha'}(\mathbf{k}),$$
 (11)

where N is the number of electrons and $a_{\alpha\alpha'}(\mathbf{k}) = (e/mv) \int_v u_{\mathbf{k}\alpha}(r) (\hbar k_z - i\hbar \partial/\partial z) u_{\mathbf{k}'\alpha'}(\mathbf{r}) d^3r$ with v being the volume of the unit cell. Thus, the conductivity can be obtained as

$$\sigma_{zz} = \operatorname{Re}\left\{ C \sum_{\mathbf{k}' \alpha', \mathbf{k} \alpha} \frac{[f_0(\varepsilon_{\mathbf{k}' \alpha'}) - f_0(\varepsilon_{\mathbf{k} \alpha})]}{\varepsilon_{\mathbf{k}' \alpha'} - \varepsilon_{\mathbf{k} \alpha} - i\hbar/\tau} \times \frac{4\pi Z e^2}{\varkappa^2 N_e} N_{imp} a_{\alpha \alpha'}(k_z) \right\},$$
(12)

where N_e and N_{imp} are the concentrations of impurities and electrons, respectively.

Supposing $\varepsilon_{\mathbf{k}'\alpha'} - \varepsilon_{\mathbf{k}\alpha} = \hbar \omega_c$, at $k_z = k_{zF}$ (where k_{zF} is the Fermi wave number) we finally obtain

$$\sigma_{zz} = C \frac{2\pi Z e^2 N_{imp}}{\kappa^2} a_{\alpha\alpha'}(k_{zF}) \frac{\tau}{1 + \omega_c^2 \tau^2} \tanh \frac{\hbar \omega_c}{2kT}.$$
 (13)

This model possesses all the essential features of the electron system considered here. Clearly, the conductivity decreases with increase of the magnetic field due to the increase of the inter-Landau-level separation ($\hbar \omega_c$). Moreover, as follows from Eq. (13), at $N_{imp} = N_e$ the conductivity increases with increasing electron density as $N_e^{2/3}$.

III. CHARACTERIZATION OF THE SAMPLES

In order to find the VLMR predicted above, we investigated structures consisting of a 20-period SL with 224 Å [80 monolayers (ML)] of GaAs wells and 8.4 Å (3 ML) of Al_{0.3}Ga_{0.7}As barriers grown by molecular-beam epitaxy on a (100) n^+ -type GaAs substrate. The SL's were doped with Si to achieve a desirable position of the Fermi level (above a miniband); they were sandwiched between two highly Sidoped 1000 Å thick GaAs contact layers. One high mobility undoped SL with the intrinsic electron concentration N_e $\approx 5 \times 10^{15} \,\mathrm{cm}^{-3}$ was studied as well. The resulting structures were etched to yield mesas with a diameter of 0.5 mm. The Ohmic contacts were prepared by depositing Au-Ge-Ni alloy annealed at 450 °C for 120 s. In order to measure the thermostimulated current, a Schottky contact was formed at the top of the superlattices by the deposition of a 100 nm thick gold layer; these structures were used to control the electron concentration by CV measurements.

In addition, a structure consisting of a doped $GaAs/Al_{0.3}Ga_{0.7}As$ 40-period SL with the thicknesses of the wells and barriers equal to 210 Å (75 ML) and 8.4 Å (3 ML), respectively, grown on a semi-insulating (100) GaAs substrate without a highly doped cap layer was used to fabricate

a Hall bar in order to measure the effect of a magnetic field on the in-plane transport. This superlattice was doped up to $N=5\times10^{17}$ cm⁻³, which corresponds to a Fermi level position well above the miniband, and it revealed a Hall mobility measured at T=10 K of about 1500 cm²/V s, which corresponds to the value $\hbar/\tau \approx 10$ meV. Therefore, low field conditions ($\omega_c \tau \leq 1$) hold in the doped superlattices with electron concentrations $N_e = (1-5) \times 10^{17}$ cm⁻³, while the high field condition ($\omega_c \tau > 1$) is expected in the undoped superlattice.

Calculations made using the envelope function approximation including the effect of nonparabolicity showed that the SL's under investigation have a 4 meV wide lowest miniband. In all these narrow miniband SL's the Fermi levels were located in the minigap: well above the miniband in the doped SL's and close to the top of the miniband in the undoped SL. Measurements of the magnetoresistance were carried out at temperatures 1.5–4.2 K in magnetic fields up to 12 T.

Differently doped wide miniband $(GaAs)_{17}(AlAs)_2$ and $(GaAs)_{17}(AlA_3)_3 Ga_{0.7}As)_6$ superlattices (with the width of the miniband $W_{SL} \approx 65$ meV) grown without the contact layers, where partially and completely occupied minibands (which correspond to closed and open Fermi surfaces, respectively) can easily be achieved, were studied in order to confirm the influence of the shape of the Fermi surface on the VLMR.

As mentioned above, the peculiar shape of the Fermi surface of a SL is responsible for the VLMR discussed here. In order to confirm that there were no contributions of the effects of the localization of electrons to the measured VLMR (similar to those observed in Ref. 12), we explored the temperature dependence of the current across the SL's. Contrary to the activation character of the transport found in Ref. 12, a decrease of the current with increasing temperature caused by the scattering of electrons by phonons was observed in all the samples studied here. In addition, measurements of the thermostimulated current (TSC), a well-known method for detecting localization of carriers in semiconductors, revealed no presence of trapping. The results of these measurements are plotted in Fig. 2; they allowed us to conclude that in the samples under investigation the transport of electrons has Bloch miniband character. Therefore, in a strong enough magnetic field we expected to find VLMR caused by the quantization of the electron energy along the layers in the SL with an open Fermi surface.

In order to confirm the shape of the Fermi surface of the SL's studied here, we measured the transverse magnetoresistance, which is known to exhibit a strong variation when the orientation of the magnetic field is changed relative to the axis of the undulating cylinder (the SL axis). In the case of a strong magnetic field the magnetoresistance for such a Fermi surface was calculated in Ref. 18 and it takes the form

$$\rho = \frac{\beta_1 B^2 \cos^2 \varphi}{\theta^2 B^2 + \lambda^2 B_0^2} C(\eta) + A, \qquad (14)$$

where φ and θ are the angles between the current direction and the *x* axis (which lies in the plane of the magnetic field) and between the cylinder axis and the plane of the magnetic



FIG. 2. Temperature dependence of the current across the $(GaAs)_{80}(Al_{0.3}Ga_{0.7}As)_3$ superlattice measured at the voltage V = 1 V (a) and of the thermostimulated current (b). The thermostimulated current was measured at different values of the reverse voltage (shown in the figure) applied to the Schottky contact fabricated at the top of the superlattice.

field, respectively, A, β_1 , and λ are smooth functions of the angles, $C(\eta)$ is a smooth function of its argument $\eta = (\omega_c \tau \theta)^{-1}$, with $C(0) = C(\infty) = 1$, and the magnetic field B_0 is associated with the condition $\omega_c \tau = 1$. According to the formula (14), in the singular direction $\theta = 0$ (open orbits) the resistivity depends on the magnetic field as B^2 , whereas in all other directions (closed orbits) it saturates at the field $B \simeq B_0/\theta$.

The vertical magnetoresistances measured in the undoped superlattice with different orientations of the magnetic field are shown in Fig. 3. In this case, even at relatively weak magnetic fields, the high field conditions hold $(B>B_0)$ and the quadratic dependence of the transverse magnetoresis-





FIG. 4. Dependence of the resistance R_{zz} across the doped (GaAs)₈₀(Al_{0.3}Ga_{0.7}As)₃ superlattice with $N_e = 1.4 \times 10^{16} \text{ cm}^{-3}$ on magnetic fields with different orientations relative to the superlattice surface measured at T = 1.5 K. The dashed line shows the calculated VLMR.

tance (the curve with $\theta = 0^{\circ}$) is revealed. However, a small but finite electron scattering actually results in $\theta \neq 0$, which causes the saturation of the transverse magnetoresistance observed experimentally. The dashed line in Fig. 3 shows the best fit obtained from the formula (14) with the scattering angle $\theta \approx 30^{\circ}$. This result confirms that the shape of the Fermi surface in the superlattices studied here is of the undulating cylinder type considered above.

IV. RESULTS AND DISCUSSION

A typical dependence of the resistance measured across the doped SL as a function of an applied magnetic field with different orientations is shown in Fig. 4. In a transverse magnetic field (normal to the z direction) we observed a weak positive VTMR. The Shubnikov–de Haas oscillations are clearly seen in this case. Their period did not depend on the doping of the SL's; therefore, they were attributed to the highly doped contact layers. Thus, we conclude that both the SL and the contacts contribute to the observed VTMR.

An increase of the positive vertical magnetoresistance was observed on increasing the angle θ between the direction of the applied magnetic field and the surface of the SL. This increase of the magnetoresistance was accompanied by a decrease of the amplitude of the Shubnikov-de Haas oscilla-



FIG. 5. Dependence of the resistance R_{xx} along the layers of the doped (GaAs)₇₅(Al_{0.3}Ga_{0.7}As)₃ superlattice with $N_e = 2 \times 10^{17}$ cm⁻³ on magnetic fields parallel and perpendicular to the growth direction measured at T = 1.5 K.



FIG. 6. Dependence of the resistance R_{zz} across the doped wide miniband superlattices GaAs/Al_xGa_{1-x}As with N_e =5.0 ×10¹⁷ cm⁻³ (closed Fermi surface) (a) and N_e =1.7×10¹⁸ cm⁻³ (open Fermi surface) (b) on magnetic field measured at *T*=1.5 K.

tions, which disappeared with the magnetic field parallel to the growth direction, when the measured VLMR originated completely from the SL, while the contact regions did not contribute. In this case the longitudinal magnetic field, quantizing the motion of electrons parallel to the layers, eliminates the contribution of the open Fermi surface to the vertical conductivity. The dependence of the resistance presented in Fig. 4 by the dashed line was calculated according to Eq. (13) as a value equal to σ_{zz}^{-1} with $\hbar/\tau=12$ meV, and it agrees well with the experimental data.

The effect of the magnetic field on the in-plane motion of electrons occupying the open Fermi surface of a SL is demonstrated in Fig. 5. As expected, the longitudinal magnetic field revealed no significant effect on the in-plane transport, while the transverse magnetic field caused a strong positive magnetoresistance accompanied by Shubnikov-de Haas oscillations, now originating from the SL.

The dependence of the VLMR on the shape of the Fermi surface was explored in the wide miniband SL's where $W_{SL} > \hbar/\tau$ and therefore the SL with Fermi level located inside the miniband (closed Fermi surface) can be easily distinguished from the SL with Fermi level above the miniband (open Fermi surface). The experimental results plotted in Fig. 6 clearly show the absence of the VLMR in the SL with the closed Fermi surface and a strong VLMR in the case of the open Fermi surface. These SL's were grown without the contact layers, which therefore did not contribute to the measured magnetoresistance.

Good agreement was found between the experimental (data points) and calculated (solid line) dependencies of the



FIG. 7. Dependence of the relative vertical longitudinal magnetoresistance in the $(GaAs)_{80}(Al_{0.3}Ga_{0.7}As)_3$ superlattices on the electron concentration measured at T=1.5 K.

relative vertical longitudinal magnetoresistance $[R_{zz}(B = 12 \text{ T}) - R_{zz}(0)]/R_{zz}(0)$ on the concentration of the electrons, plotted in Fig. 7.

As a matter of fact, in a SL subject to a strong enough magnetic field perpendicular to the surface (such as $\hbar \omega_c$ $>\hbar/\tau$), as a consequence of the vanishing vertical conductivity, the state of the three-dimensional electron gas resembles the quantized Hall state of a two-dimensional electron gas. This means that in appropriate conditions such a magnetic field forces the three-dimensional electron system, where it is not possible to observe the QHE, to acquire a state favorable for the observation of the QHE. It should to be stressed that, although the nature of the VLMR observed here seems to be similar to that of the OHE, there is an essential difference: the quantization of the in-plane electron energy is responsible for the QHE, while both the quantization and the specific shape of the Fermi surface of the superlattice determine the VLMR. As we observed, the VLMR does not depend as strongly on temperature as do the QHE and the conductivity caused by chiral surface states; therefore, it was detected at rather high temperatures when no signs of the QHE or surface states were found.

V. CONCLUSIONS

To conclude, we found vertical longitudinal magnetoresistance originating from the unusual shape of the Fermi surface of a semiconductor superlattice. It was shown that the quantization of the in-plane motion of electrons is responsible for the observed effect. The absence of the VLMR in superlattices with closed Fermi surfaces and when the current was measured parallel to the layers confirmed this conclusion.

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