Hall effect in a spatially fluctuating magnetic field with zero mean

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We study the Hall effect of a nonplanar stripe-shaped two-dimensional electron gas (2DEG) with stripes oriented perpendicular to the current in the presence of an in-plane external magnetic field B_{ext}. Since the 2DEG is sensitive only to the normal component of B_{ext}, the electrons move in a sign alternating magnetic field with zero mean $\langle B\rangle=0$ and nonzero variance, which is proportional to B_{ext}. A zero Hall resistance is observed at low B_{ext}, as expected for $\langle B\rangle=0$. However, for $B_{ext}>5$ T, a nonzero Hall voltage is found. In this regime the electron transport is characterized by the propagation of chiral snakelike trajectories. These states move in positive and negative directions along stripes and are confined to one-dimensional channels with a different number of propagating modes, which is determined by the surface topography. Asymmetry in the number of ballistic modes leads to a nonzero Hall voltage.

Transport properties of a two-dimensional electron gas (2DEG) in a spatially fluctuating magnetic field with zero average have been a subject of much experimental and theoretical works. The interest is motivated by its relevance to the localization problem in random fields and the fractional quantum Hall effect. Zero average magnetic field leads to a zero Hall conductivity. Therefore, it was argued that the electron states are localized in a random magnetic field with zero mean,[1] because the criterion for the creation of the extended states is that the classical Hall conductivity must approach the value of the quantum conductance.[2] This conclusion is contradicted by calculations that demonstrate that the local Hall conductance can be nonzero for a specific random flux configuration because of the breaking of time reversal symmetry.[3] Experimentally anomalous Hall resistivity in random magnetic field has been studied in heterostructures with 2DEG covered by superconducting films.[4]

The main features of the transport properties of a 2DEG in a random magnetic field with zero mean has been demonstrated most distinctly in systems with smooth fluctuations. Within this approach, the correlation radius of the magnetic-field fluctuations is larger than the typical magnetic length. In this limit, the electron motion can be described semiclassically.[5] As was first noted by Muller in Ref. 6, where the amplitude of the magnetic field is small, there exist classical trajectories— so-called snake states that cross the $B=0$ line. These states propagate perpendicularly to the field gradient ∇B and have a time-reversal asymmetry, which is illustrated in Fig. 1 for periodical sign alternating magnetic field. When $\nabla B>0$, snake states travel in the positive y direction, and for $\nabla B<0$ these trajectories carry probability current in the negative y direction. In the region where magnetic field is large, the electron follows rapid cyclotron orbits that slowly drift along contours of constant field (not shown). These trajectories have a mean velocity in the opposite directions to the snakelike orbits. Therefore, the net current in equilibrium is zero.

In random B with field distribution, which is symmetric

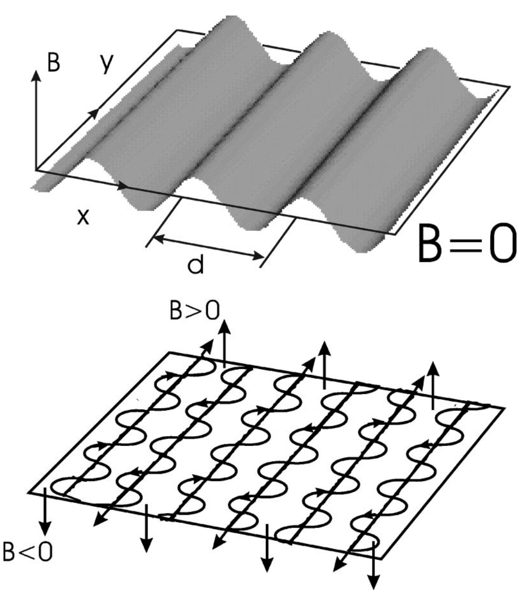

FIG. 1. Schematic view of the periodic magnetic field and classical picture of the electron trajectories subjected to this field.

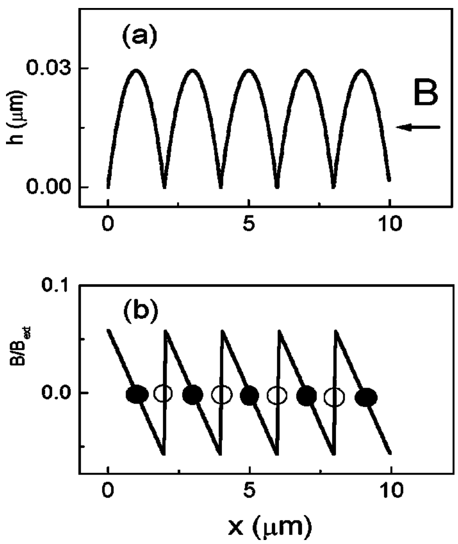

FIG. 2. Profile of the surface of two-dimensional electron gas and (a) consistent magnetic field profile (b) for the external magnetic field oriented parallel to the *x* axis. Full circles- position of the snake-trajectories channeling in the positive *y* direction, open circles- snake states running in the negative *y* direction.

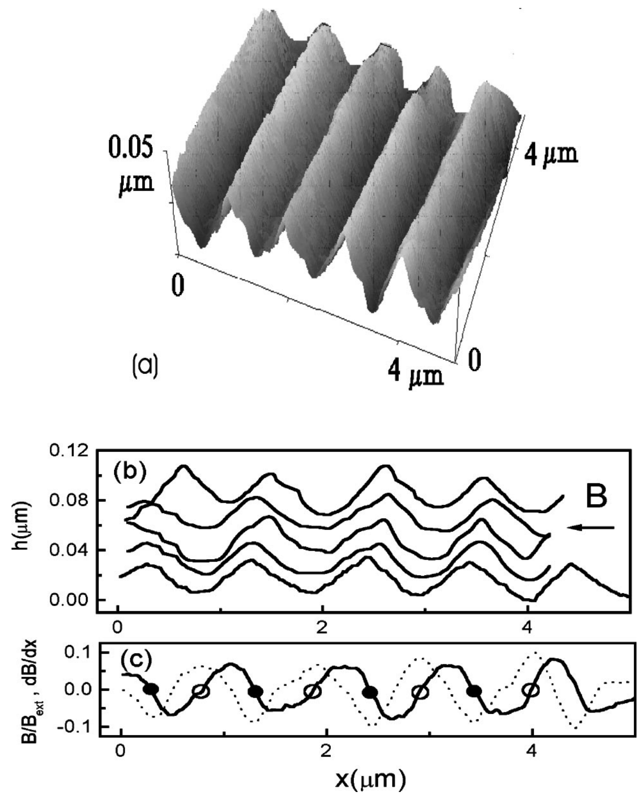

FIG. 3. Atomic force microscope image of the sample surface. (b) Profiles of the sample surface across the stripes from the different parts of the sample. Curves are shifted for the clarity. (c) Solid line-magnetic-field profile calculated for the surface profile (b-bottom curve) when magnetic field is oriented parallel to the *x* axis and perpendicular to the stripes, dots- gradient of the magnetic field. Full circles- position of the snake-trajectories channeling in the positive *y* direction, open circles- snake states running in the negative *y* direction.

about the zero mean value $\langle B\rangle = 0$, the electron motion consists of percolating snakelike trajectories along contour $B = 0$ closed around hills or valleys of the magnetic-field surface with tunneling near saddle points.[5] This situation can be represented as links on a square network, carrying a fixed number of the electronic snake states. The drift trajectories are localized and do not contribute to the conductivity. Within this approach the network consists of the plaquettes of the lattice with clockwise and anticlockwise circulation. As was indicated in Ref. 5 the mean Hall conductance of the system is zero, because these plaquettes are statistically equivalent. An imbalance of the clockwise and anticlockwise circulation, which can occur locally because of the random character of the potential, should lead to nonzero local Hall conductivity. For the sake of simplicity consider the situation, when the average cyclotron radius R_c is smaller than half of the magnetic-field periodicity *d*, but the amplitude of the magnetic field is sufficiently small so that $d/4 < R_c < d/2$. In this regime there are no drifts orbits, and the transport is determined by the snake trajectories as shown in Fig. 1. In this situation, we demonstrate another possibility to obtain nonzero Hall current for $\langle B\rangle = 0$. We create snake states using periodical sign-alternating magnetic field. Recently 2DEG systems grown on a nonplanar prepatterned GaAs substrate have been used to produce essentially inhomogeneous magnetic fields.[7,8] Since orbital motion of the 2DEG is sensitive to only the normal component of *B*, electrons in such systems experience sign alternating magnetic field with zero average, when uniform field is applied parallel to the substrate. Consider a 2DEG regrown over a periodical array of the stripes with profile across the array (*x* direction) as shown in Fig. 2(a). We see, that the top of the stripes is flat in contrast to the sharp steps between stripes. For magnetic field oriented parallel to the substrate in the *x* direction, the normal component of *B* can be expressed as $B_N = (\nabla B_{ext}\nabla f)/|\nabla f| \approx (df/dx)B_{ext}$ where gradient $\nabla f(df/dx, df/dy, df/dz)$ is defined for the surface $f(x,y,z) = 0$. If the height of the stripes h is smaller than periodicity, $df/dx \approx h/d \ll 1$, and we have $B_N \approx (h/d)B_{ext}$. Figure 2(b) shows the magnetic-field modulation for the surface profile corresponding to Fig. 2(a). Snake states running in the positive and negative directions *y* are marked by open and full circles. We see, that these trajectories travel in very different field gradient, and, therefore, have a different number of propagating modes.[6] Because the stripes are oriented in the direction perpendicular to the current probes, snake trajectories contribute directly to the Hall effect. Thus, the net current in such structures is unbalanced, and nonzero Hall voltage appears.

Samples were fabricated employing overgrowth of GaAs and $Al_xGa_{1-x}As$ materials by molecular beam epitaxy on the prepatterned (100) GaAs substrate. Details of the sample preparation are reported in Ref. 9. Figure 3 shows the atomic force image and profile of the surface in the *x* direction. We see that the surface consists of a periodical array of ridges. The average periodicity of the ridges is 1 μm, and the corrugation height *h* is 300 Å. We note that the array is more regular, and corrugation height is larger than in the samples studied previously in Ref. 9. After regrowth samples with the patterned area were processed into Hall bar, with the nonplanar surface situated on one side of the Hall bar, as shown in Fig. 4(a). The distance between voltage probes was 100 μm

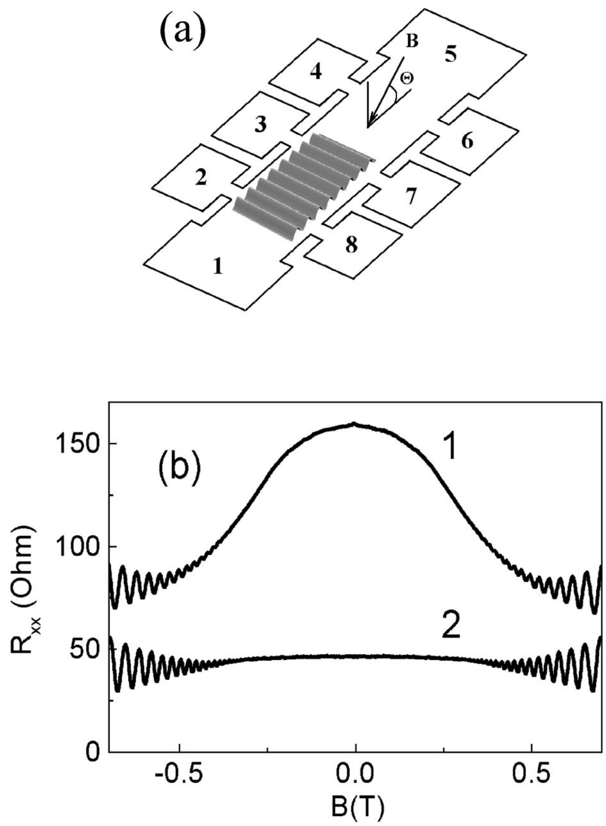

FIG. 4. (a) Schematic view of the sample and experiment geometry. (b) magnetoresistance as a function of B for the field perpendicular to the substrate plane, 1- resistance measured between voltage probes on the nonplanar region (voltage probes 2-3, current probe 1-5), 2- resistance measured between voltage probes on the planar region (voltage probes 3-4, current probe 1-5), $T=1.5$ K.

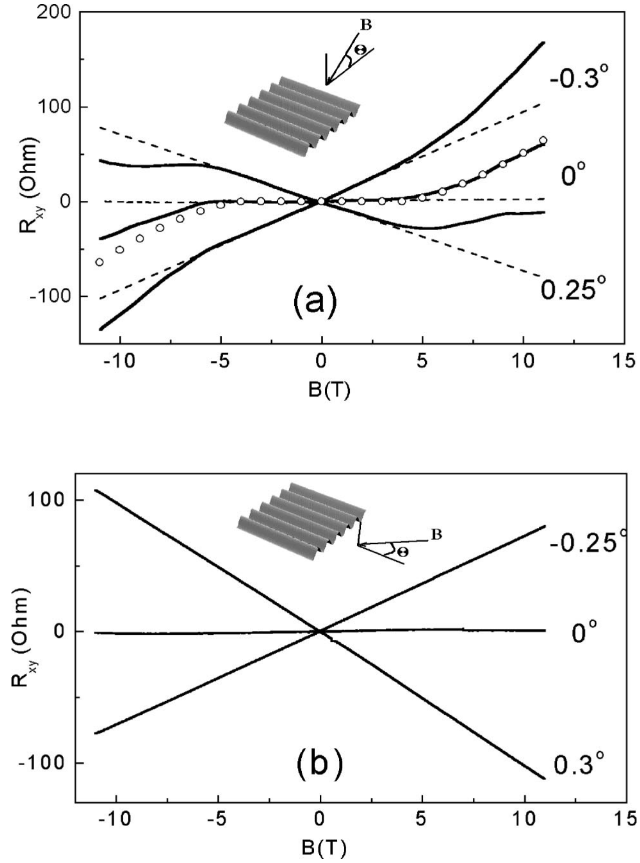

FIG. 5. Hall resistance as a function of B for different angles between the applied magnetic field and substrate plane. (a) magnetic field is oriented perpendicular to the stripes, (b) magnetic field is oriented parallel to the stripes. Solid traces- resistance measured between voltage probes on the nonplanar region (voltage probes 2-8, current probes 1-5), dashes- resistance measured between voltage probes on the planar region (voltage probes 4-6, current probe 1-5), $T=1.5$ K. Circles- fitting to the equation $\rho_{xy} \sim (B_{ext}\text{-}B^*)^{3/2}$, $B^*=4$ T.

and the width of the bar was 50 μm. The mobility of the 2DEG in the planar part of the sample is 550×10^3 cm^2/Vs, and electron density $n_s=5\times 10^{11}$ cm^{-2}. The measurement temperature was 1.5 K. We study 3 samples with identical parameters, which demonstrate similar results. The magnetoresistance of a typical specimen is shown in Fig. 4(b) with the magnetic field oriented perpendicular to the substrate. We see that the resistance at $B=0$ of the patterned part of the Hall bar is 4 times larger than the resistance of the planar 2DEG. Note also the large negative magnetoresistance of the nonplanar structure. However, at high-magnetic fields the Shubnikov de Haas (SdH) oscillations are almost identical for both the nonplanar and the planar 2DEG. It is worth noting here that in the samples with the ridges oriented parallel to the current, studied in Ref. 9, the mobility of the electron gas in the patterned and the nonpatterend part of the Hall bar is the same. We attribute this mobility anisotropy to scattering by the corrugated GaAs/Al$_x$Ga$_{1-x}$As interfaces. When electron travels from one facet to another, the vertical component of the electron Fermi vector is changed. Thus, the electron experiences scattering similar to the scattering by the interface roughness. Anisotropic conductivity and large negative magnetoresistance for lower mobility has been observed in a 2DEG grown on a (311)B GaAs substrate.[10] Scattering by the periodic arrays of the terraces on the vicinal substrates has been assumed to explain the effect of the mobility anisotropy. Hall resistances between probes 2-8 (nonplanar region) and 4-6 (planar region) in perpendicular external magnetic field are found to be essentially identical for both parts of the sample. Figure 5(a) shows the results of the Hall effect measurement, when the applied magnetic field is exactly parallel to the substrate, with the in-plane component perpendicular to the ridges and with the magnetic field slightly tilted away from the substrate plane. We see that in the presence of the in-plane field the Hall resistance of the planar 2DEG is zero. Surprisingly, the Hall voltage of the corrugated 2DEG, which is also equal to zero at low field, appears at $B>5$ T and starts to grow almost linearly with B. The Hall voltage changes sign for reverse B polarity, which gives us confidence that the probes are aligned, and that anomalous behavior of the Hall effect is not due to the mixing of the magnetoresistance component. However, the small asymmetry upon reversing B can be explained by the influence of the longitudinal resistance. Hall voltage of the planar 2DEG is exactly zero, which confirms that the magnetic field has only component parallel to the substrate, and, therefore, spatially fluctuating magnetic field in the corrugated part of the samples has a zero mean. For small tilt angles the Hall voltage of the nonplanar 2DEG still deviates from the linear

B-dependence. We even find that at angles $0^o < \Theta < 0.25^o$ the Hall constant can change sign at strong magnetic field and therefore has a sign corresponding to two-dimensional holes rather than for electrons. Figure 5(b) shows the measured Hall resistance, when magnetic field is oriented parallel to the stripes. We see that the Hall voltage is zero at $\Theta = 0^o$, and varies linearly with B at $\Theta \neq 0^o$. It is also identical to the Hall voltage of the planar 2DEG in the tilted field when we expect homogeneous magnetic field. We, therefore, conclude that the nonzero Hall resistance is due to the strong spatially fluctuating magnetic field with zero mean. We attribute such anomalous Hall effect to the asymmetry of the chiral snake states propagation.

For the small applied magnetic field the average cyclotron radius R_c is larger than the periodicity. In this situation, electrons experience chaotic motion similar to the motion in the periodic antidot lattice at low magnetic field.[11] In high magnetic fields, when $R_c < d/2$ (see Fig. 1) snakelike trajectories are formed. Considering a realistic profile of the surface, as shown in Fig. 3(b), we find that the amplitude of the magnetic field fluctuations approaches 8% of the external field [Fig. 3(c)]. For $B_{ext} \approx 5$-6 T it corresponds $B_N^{max} \approx 0.4$-0.5 T, and cyclotron radius approaches 0.25-0.3 μm. Taking into account that the average magnetic field is smaller than amplitude of the effective magnetic field: $B_N \approx B_N^{max}/2$, we have $R_c \approx 0.5$-0.6 $\mu\text{m} \approx d/2$ in agreement with the value of the magnetic field, for which the anomalous Hall voltage appears [Fig. 5(a)]. It is worth noting here, that we also measure diagonal resistance in the presence of the in-plane external magnetic field. When the in-plane component is perpendicular to the stripes, the resistance across the ridges increases from 160 Ohm at $B = 0$ T to 24000 Ohm at $B = 11$ T, a ratio of 150. Such large positive magnetoresistance has been studied in Ref. 7 and explained by the formation of the magnetic barrier across the current flow.[12] At $B_{ext} < 5$ T we find that magnetoresistance varies quadratically with the magnetic field, however, for larger magnetic fields a linear dependence $R \sim (B_{ext} - B^*_{ext})$ of magnetoresistance is observed, where $B^*_{ext} \approx 4$-5 T. We assume that this behavior demonstrates a crossover from the weak magnetic-field fluctuations, when transport is chaotic, to the strong fluctuations regime, where regular snakelike trajectories exist. Magnetoresistance in this regime has been measured and analyzed in Ref. 13 for periodical magnetic field with a nonzero average. We thus conclude that for $B_{ext} > 5$ T the amplitude of the magnetic field fluctuations is large enough to bind electrons between maxima and minima of the field and create snakelike trajectories propagating along $B_N = 0$ line. We have to note also, that the potential distribution in the sample in our case is the same as considered for the classical transport of electrons through magnetic barriers in series.[12] The current is effectively injected into the corner of the first magnetic barrier, carried by the snake trajectories and after removed from the narrow region of the diagonally opposite corner. For the second magnetic barrier the injection point of the current is at the same side as a removal point from the first barrier. The current in the second magnetic barrier is carried out by the snake trajectories propagating in the opposite directions. The process can be continued for the multiple barriers in series. If the current in the second barrier is not equal to the current in the first barrier, nonzero Hall voltage appears. Indeed electron charge cannot be accumulated, and the excess of the Hall current is compensated by the current between barriers at the same side as an injection point of the current into the first barrier. When $d/4 < R_c < d/2$ coupling between electron states via tunneling through the magnetic barriers in series should be taken into acount. However, we believe that due to the fluctuations of the barrier heights and periodicity, magnetic superlattice effects are unlikely to be observed in our system. The large positive magnetoresistance supports this assumption. Thus we can consider electron motion in a single magnetic barrier. In this case, in a constant gradient field the effective magnetic field can be written as $B_N = -\alpha B_{ext} y/d$, where $\alpha \approx h/d$. For the Landau gauge $B = -dA_x/dy$, the Schrodinger equation for the transverse motion is

$$1/2\{-2\Lambda^2 d^2/dy^2 + [k\Lambda - y^2/2\Lambda^2]^2\}^2 \phi = [E_n(k)/E_\Lambda]\phi$$

where wave function is factorized as $\psi = e^{ikx}\phi(y)$, $\Lambda = (h/e\nabla B)^{1/3}$ and $E_\Lambda = \hbar^2/m\Lambda^2$. For large negative $k\Lambda$ the confining potential is a single well with effective potential $V_{eff} = E_\Lambda(k\Lambda - y^2/2\Lambda^2)^2/2$.

The eigenstates for this potential corresponds to the snake trajectories. The snake states form a bundle of the width given by[14]

$$W = \Lambda(2k_F\Lambda)^{1/2}, \tag{1}$$

where k_F is the Fermi wave vector. In accordance with the Landau-Buttiker formalism the conductance of the ballistically propagating modes is given by

$$\sigma = (2e^2/h)(k_F W/\pi). \tag{2}$$

Substituting Eq. (1) in Eq. (2), we find

$$\sigma = (2^{3/2}e^2/\pi h)(k_F\Lambda)^{3/2} \sim \nabla B^{-1/2}. \tag{3}$$

If ∇B_+ for states running in positive y direction is larger than ∇B_- for trajectories running in negative y directions

$$\sigma_+ - \sigma_- \sim \nabla B_+^{-1/2} - \nabla B_-^{-1/2} \neq 0.$$

For all investigated samples we find a positive Hall voltage at $B > 0$. The sign of the Hall effect determines the direction of the excess charge flow. We find that states channeling along the top of the stripes have a larger number of ballistic modes and, consequently, a larger conductance. From the realistic profile of the surface, shown in Fig. 3, we find the magnetic field gradient $\nabla B \approx 4G/\text{Å}$ at $B_{ext} = 10$ T. From this we obtain the value of the band width $W = 0.6$ μm, and number of the snake states $k_F W/\pi = 30$. The patterned area of the sample was 140×140 μm^2, therefore we have $N = 140$ channels for the positive and the same number for the negative y direction. The total number of the snake modes channeling in one direction is therefore $S \approx 140 \times 30 = 4200$. To obtain the value of the Hall conductance we have to convert the conductivity tensor to a resistivity tensor. If the Hall conductance is small, we have

$$\rho_{xy} = \sigma_{xy}/(\sigma_{xy}^2 + \sigma_{xx}^2) \approx \sigma_{xy}\rho_{xx}^2. \tag{4}$$

From the experiment we find $\sigma_{xy}=2\times10^{-3}e^2/h$ at $B=10$ T. Now, we can obtain the coefficient of the asymmetry between snake states channeling in the positive and the negative y directions. From the comparison of the Eq. (2) and experimental value, we find that only 8 channels or 6% from the total number N can carry one additional snake mode. From the Fig. 3 we can see that the array of the stripes has both an irregular and periodical component, therefore the number of propagating modes fluctuates from one stripes to another. Taking the difference in the gradients $10-20\%$, we obtain the deviation of the snake states number from the average value $\sim$2-3. Therefore, the nonzero Hall conductance in our case can arise from the incomplete statistical averaging of the number of states channeling in positive and negative y directions. We have to emphasize here, that the average magnetic field is exactly zero in our case, and nonzero Hall effect arises from the difference in the magnetic field gradient due to the topography of the periodical magnetic field, as shown in Fig. 2. Indeed in realistic samples we have no such periodical magnetic field, and resulting Hall conductance is small because of the averaging of the corrugations height fluctuations. It is also worth to note here, that we consider ballistic case, however the scattering of the snake trajectories should be taken into account. Two scattering mechanisms can be considered—impurity scattering and reflection of the snake states by magnetic-field irregularities. Note that the probability of the impurity scattering is decreased in our situation at $B_{ext}\sim10$ T. As pointed out by Buttiker in the quantum Hall effect regime[15] edge states scattered by the impurity continue to follow the edge. For the snake states when $R_c<$d/4 the electron can change between $B=0$ contour lines, only if the impurity is located in the middle between the $B=0$ lines. Another mechanism can be a scattering by the magnetic field irregularities. As we find from the atomic force microscope image, array of ridges have both an irregular and a periodical component. For example, the height of the ridges strongly fluctuates (30%) in the y direction. This irregularity leads to fluctuations in the magnetic field gradient along the zero field lines. If the height of the corrugation is smaller than the average height, the cyclotron diameter becomes larger than $d/2$, and the electron can change zero line contours. If the ballistic transport is partially suppressed, we can assume that the coefficient of the asymmetry between snake states channeling in the positive and negative directions is smaller than $3-5\%$. This value can be estimated from the precision of the atomic force microscope measurements. Such asymmetry can arrise from the growth process. The difference in the crystal growth rate on the maxima and minima and smoothing of the steps in maxima can lead to profile, which produce different field gradient on the top of the ridges and in the region between stripes. In this case we can obtain $\sigma_{xy}\approx2e^2/h$. Assuming that ballistic transport is suppressed by the damping factor exp($-L/l$), where L is the width of the sample, l is the mean free path of the snake trajectories, we find from the comparison of the experimental and expected σ_{xy} values that $l=7-8$ μm. This value is comparable with mean free path due to the scattering by impurities in zero-magnetic field. Numerical simulation of the transport in irregular stripe-shaped 2DEG is necessary to clarify this question.

Thus, the asymmetry of the snake states propagation arises from the topography of the 2DEG, as shown in Fig. 2. The strong suppression of the Hall effect in comparison with expected value can be due to the scattering of the snake states or, in the ballistic case, due to the averaging of the difference in the gradient for the positive and negative y directions. We belive that the ballistic transport is unlikely to be observed in our macroscopic samples (50 μm width), therefore the nonzero Hall effect is mainly due to the preferential crystal growth leading to the asymmetry in the 2DEG topography.

Finally we examine the B dependence of the observed Hall effect. As mentioned above, we find $\rho_{xx}\sim(B_{ext}$-$B^*_{ext})$. From Eq. (2) $\sigma_{xy}\sim\nabla(B_{ext}$-$B^*_{ext})^{-1/2}$, because the snake states are found only when $B>B^*_{ext}$. Substituting this expression into Eq. (4) we obtain $\rho_{xy}\sim(B_{ext}$-$B^*_{ext})^{3/2}$. Figure 5(a) shows this dependence and we see that the agreement is good.

In conclusion, we have observed a nonzero Hall resistance of a two-dimensional electron gas in a sign alternating magnetic field with zero mean. Naively, this observation disagrees with the conclusion that the random magnetic field should lead to a zero Hall conductance. However, we demonstrated that the asymmetry in the propagation of the snake-like trajectories could lead to a nonzero Hall voltage. This asymmetry arises from the difference in the field gradient for different chiral snake trajectories. Therefore, it will allow us to construct specific class of the random magnetic field with zero mean with hidden asymmetry between chiral states. As the nonzero Hall conductance is responsible for the delocalized properties of the electron wave functions, electron states will be extended in these random magnetic fields.

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