Quantum interference effects in a strongly fluctuating magnetic field

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We have studied the negative magnetoresistance and universal conductance fluctuations (UCF's) in a two-dimensional electron gas (2DEG) grown on substrates with prepatterned, submicrometer dimples. Since the 2DEG is sensitive only to the normal component of B, electrons move in a spatially inhomogeneous (B perpendicular to the substrate: $B_{\perp}$) or sign-alternating random magnetic field (B parallel to the substrate: $B_{\parallel}$). Quantum interference effects in a random magnetic field due to the magnetic flux appear only due to second-order effects. The UCF's are found to be 1.5–2 times smaller for $B_{\parallel}$ than for $B_{\perp}$. [S0163-1829(96)09419-2]

The localization problem for a two-dimensional electron gas (2DEG) experiencing a random magnetic field has recently attracted considerable interest, in large part due to its relevance for the fractional quantum Hall effect.[1] It is generally accepted that a 2DEG is localized at zero magnetic field.[2] Corrections to the conductivity due to the quantum interference effects are responsible for the weak localization.[3] A magnetic field can suppress these quantum corrections,[4] but a further criterion for the creation of extended states is that the classical Hall conductivity must approach the value of the quantum conductance,[5,6] e^2/h. On the other hand, in the case of a static, random magnetic field these theories predict that a magnetic field cannot suppress the localization at all.[7] First of all, since the average magnetic field is equal to zero, weak localization corrections are not suppressed, and all states are localized as in zero magnetic field. Second, a zero average magnetic field leads to zero Hall conductivity, and hence to the absence of the topological term which is responsible for the delocalized properties of the electron wave functions. These conclusions are contradicted by numerical calculations[8] that have been used to argue that the Hall conductance for each electron eigenstate can be nonzero for a specific random flux configuration because of the breaking of time reversal symmetry, even if the total Hall conductance is zero on average. This unclear theoretical situation has yet to be elucidated by experiments. Only very recently has it been possible to realize an inhomogeneous magnetic field for 2DEG systems.[9]

Here experimental evidence is shown that, in a magnetic field, electrons confined to a nonplanar GaAs/$Al_xGa_{1-x}As$ heterojunction experience a random magnetic field B, and hence, this can be used as a model system to study the delocalization problem with random magnetic flux. Previously, we have presented high magnetic-field results demonstrating that electrons in such a system move in an effective inhomogeneous field.[10] Here we study the effect of a weak magnetic field. The results suggest that a random value of the magnetic flux is enclosed by the loops formed by the electron paths due to impurity scattering. These paths can be considered as a random walk through a periodic, sign-alternating magnetic field, giving second-order corrections to the total enclosed flux, and leading to the observed negative magnetoresistance. In small structures (2×2 μm^2), universal conductance fluctuations (UCF's) at the scale of e^2/h, due to quantum interference, were measured. We find that the correlation properties of these fluctuations in a random field are governed by the second-order corrections to the flux through the closed loops. However, the amplitude of the fluctuations is 1.5–2 times smaller than in a uniform (or slightly inhomogeneous) magnetic field.

Samples were fabricated employing overgrowth of GaAs and $Al_xGa_{1-x}As$ materials by molecular beam epitaxy (MBE) on prepatterned (100) GaAs substrates. The prepatterning consisted of lattices (periodicity $d=0.3$ and 1 μm) of holes (depth 1 μm, diameter 0.1–0.3 μm), made by electron beam lithography and wet etching. A thick (1 μm) GaAs buffer layer was grown to smooth out any steps in the crystal planes, and a rapid planarization of the initial surface is indeed seen in scanning electron micrograph of the structure surface. A ''dimpled'' surface is obtained with a modulation of the surface of $\simeq 0.1$ μm. This agrees with studies of MBE overgrowth on corrugated (100) GaAs substrates.[11] The region of etched holes is nonplanar and has a smooth slope on all edges. A scanning electron microscope (SEM) picture of this ''dimpled'' surface is shown in Fig. 1. It consists of unetched planes surrounding nonplanar valleys with planes

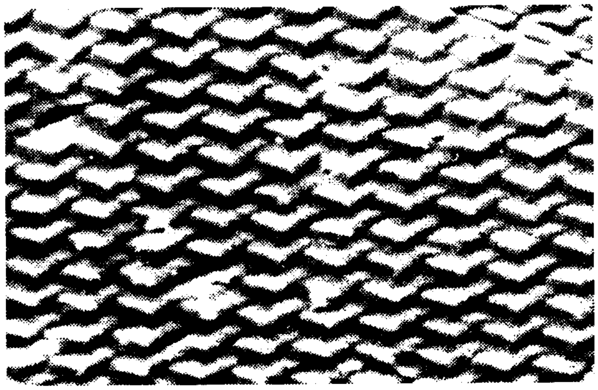

FIG. 1. A plane-view scanning electron micrograph of a dimpled sample.

tilted towards the center of the valley by $\sim 10°$ with respect to the normal of the substrate, and slopes with tilt angles 45°–60° between the valleys and the unetched surfaces. An $Al_xGa_{1-x}As/GaAs$ heterojunction was grown, with a doping setback of 100 Å and a spacer of 100 Å, to obtain a ''dimpled'' two-dimensional electron gas (dimpled 2DEG). The mobility of the 2DEG is 2.5–7 m^2/V s, and the density 5.5×10^{11} cm^{-2}. Samples with both macroscopic and mesoscopic (2×2 μm^2) dimensions have been studied. Four-terminal measurements of the magnetoresistance were carried out at temperatures $\geqslant 50$ mK.

When placed in a magnetic field, the field normal to the dimpled 2DEG is spatially modulated. Since the 2D electrons are sensitive only to the normal component of B (neglecting the spin degree of freedom), they will move in an inhomogeneous (B perpendicular to the substrate, $B_\perp$) or sign-alternating, ''tiled'' magnetic field (when B is parallel to the substrate, $B_\parallel$).[10] The electron mean-free path ℓ is 0.3–0.7 μm in the investigated samples, which is comparable to the periodicity of the magnetic field, $d = 0.3$–1 μm. Thus, in spite of the magnetic field created by the lattice of dimples being periodic, the electrons will experience a ''random'' magnetic field due to the random-walk nature of their motion imposed by the impurity disorder. In the presence of impurities, the backscattering probability of the electrons is enhanced due to quantum interference. Suppression of the interference is responsible for the low-field negative magnetoresistance.[4] In Fig. 2 this magnetoresistance is shown for the magnetic field oriented either perpendicular or parallel to the substrate. We see that magnetoresistance is strongly anisotropic, i.e., $\rho_{xx}(B_\perp)$ saturates for smaller fields than $\rho_{xx}(B_\parallel)$. The change in the conductivity with $B_\perp$ is well understood in terms of weak localization, where the magnetic field breaks the time-reversal symmetry of closed electron paths. Neglecting the effect of the small modulation in the effective magnetic field, the conductivity[4] is given by

$$\Delta\sigma_{xx}(B) = (e^2/2\pi^2\hbar) f(x), \tag{1}$$

where $x = 4eBL_\varphi^2/\hbar c$, $f(x) = \ln(x) - \Psi(1/2 - 1/x)$, and $\Psi(y)$ is the digamma function. This correction to the conductivity fits well to the data in Fig. 2, assuming a phase-coherence length $L_\varphi = 1.2$ μm at $T = 50$ mK. The linear dependence on B of the argument x of the function f simply reflects the fact that the phase shift in the interference term for the electron probability for return to the point of departure is proportional to the magnetic flux through closed loops.

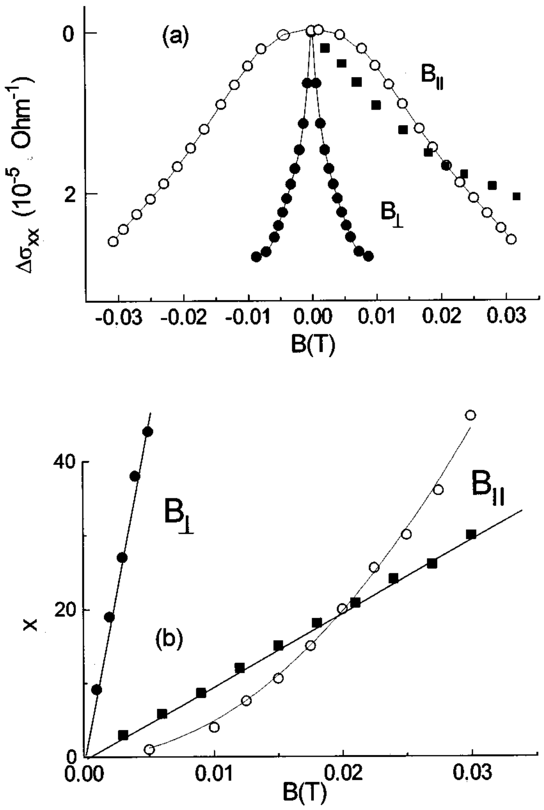

FIG. 2. (a) The low-field change in the conductance at $T = 50$ mK, as a function of magnetic field, for a dimpled sample with periodicity $d = 0.3$ μm. The full lines are the measured values for perpendicular and parallel magnetic fields. Open circles: the calculated magnetoconductance for a parallel field, using Eq. (1) and $L_\varphi = 1.2$ μm; filled circles: the calculated magnetoconductance using a B^2 dependence of the argument x in Eq. (1); filled squares: the calculated magnetoconductance using a linear B dependence in the argument x in Eq. (1). Both the open circles and the filled squares are for parallel field. (b) The argument x of Eq. (1). The markers are the same as in (a), and correspond to the values extracted from the measurement data and Eq. (1), using the values of the coefficients obtained from the fits. The full lines are guides to the eye. [The figure differs from an earlier presented version (Ref. 13) by virtue of improved analysis of the experimental data.]

The magnetoresistance in a parallel magnetic field should have the same functional behavior as Eq. (1), and the actual topological configuration of the system only enters into the argument x. In Fig. 2 it is shown that a quadratic and not a linear dependence on B in the argument fits the measured negative magnetoresistance for a parallel magnetic field. This indicates the importance of second-order effects, which may be expected since the random character of the effective field will lead to cancellation of most of the magnetic flux through the closed loops. From the theory of weak localization, quantum corrections to the conductivity can be expressed as

$$\Delta\sigma_{xx} \sim e^2/h \int \frac{dt}{t} \exp(-t/\tau_\varphi) \langle \exp(2\pi i \phi(t)/\phi_0) \rangle, \tag{2}$$

where τ_φ is a phase-coherence time, $\phi(t)$ is the magnetic flux, and $\phi_0 = hc/e$. Since in a random magnetic field the average flux is zero, it is possible to retain only the even terms in the average of the exponential: $\langle \exp[2\pi i \phi(t)/\phi_0] \rangle = \langle \cos[2\pi\phi(t)/\phi_0] \rangle \approx 1 - \frac{1}{2}\langle [2\pi\phi(t)/\phi_0]^2 \rangle + \cdots$. The next term in the sum will be an order of magnitude smaller, and is ignored. Considering a ''chess board'' magnetic field, where B changes sign in different cells, this is just a ''random-walk'' problem. The average of the square of the flux gives $\langle \phi(t)^2 \rangle \approx N\phi_N^2$, with $N = D\tau_B/d^2$ and $\phi_N = d^2 B$, where τ_B is the magnetic relaxation time, and d is the length of each square of the ''chess board.'' In analogy with a uniform magnetic field, the effect of a magnetic field is essentially to introduce a long time cutoff—the magnetic relaxation time in Eq. (2). The backscattering of an individual loop is quenched when the flux through it equals the quantum flux. In the same vein, the second-order corrections to the flux cut off the backscattering when $\frac{1}{2}\langle [2\pi\phi(t)/\phi_0]^2 \rangle \approx 1$, i.e., when $\tau_B \approx 2(hc/deB)^2/D$. In Eq. (1) x is τ_φ/τ_B (see Ref. 12), so that, in a random ''tiled'' magnetic field $x \approx (Se/2dc)^2 (B_\parallel L_\varphi)^2$. This has a square dependence on B that is observed experimentally in Fig. 2. Finally, it is necessary to look at the topology of the sample to obtain the correct area S, through which the flux is passing. As the coherence length is slightly larger than the spatial periodicity of the random magnetic field, one expects to have a total flux equivalent to that of the homogeneous field passing through an area $S = a \times b$, where a is the width of the step of the dimpled surface ($a \leqslant d/2$), and b is the height of the step. Using $S = 0.04\ \mu\mathrm{m}^2$ (a reasonable value, as seen from the SEM pictures), an excellent fit to the experimental curves in Fig. 2 is obtained.

In mesoscopic samples, quantum interference is responsible for the sample-specific universal conductance fluctuations, which have an amplitude of the order of e^2/h. It has been predicted that in a random magnetic field the amplitude of these fluctuations is still of the order of the universal value, although the correlation properties will depend on the specific realization of the random magnetic potential.[12] The magnetoresistance fluctuations of mesoscopic samples with a dimpled 2DEG were measured in perpendicular and parallel magnetic fields. Reproducible resistance fluctuations are seen for both orientations of the magnetic field. Figure 3 shows the average amplitude of the conductance fluctuations (rms) and correlation magnetic field as a function of the temperature in perpendicular and parallel magnetic fields. The rms amplitude of the fluctuations in $B_\perp$ saturates at low temperature and approaches a value $0.7e^2/h$, close to the predicted value for the 1D case.[14] This suggests that at this temperature the phase-coherence length becomes comparable to the sample size L (1.5–2 μm). This value of L_φ is close to that extracted from the weak localization measurement on the macroscopic sample (see Fig. 2). Additional evidence for L_φ being larger than the sample size at low temperatures comes from the observation that $R(-B) \neq R(B)$. This is expected for our four-terminal measurements, since the nonlocal effect becomes important when $L_\varphi \geqslant L$. The contribution to the interference pattern from the contact area (which is also a 2DEG) in four-terminal measurements leads to such magnetic-field asymmetries.[15] However, the phase-coherence length deduced from the value of the correlation magnetic field $B_c = \alpha\phi_0/L_\varphi^2$ is somewhat smaller, $L_\varphi = 0.7\ \mu$m for $\alpha = 1$. The coefficient α is probably somewhat less than unity for $L_\varphi < L_T$, where L_T is the thermal length.[14]

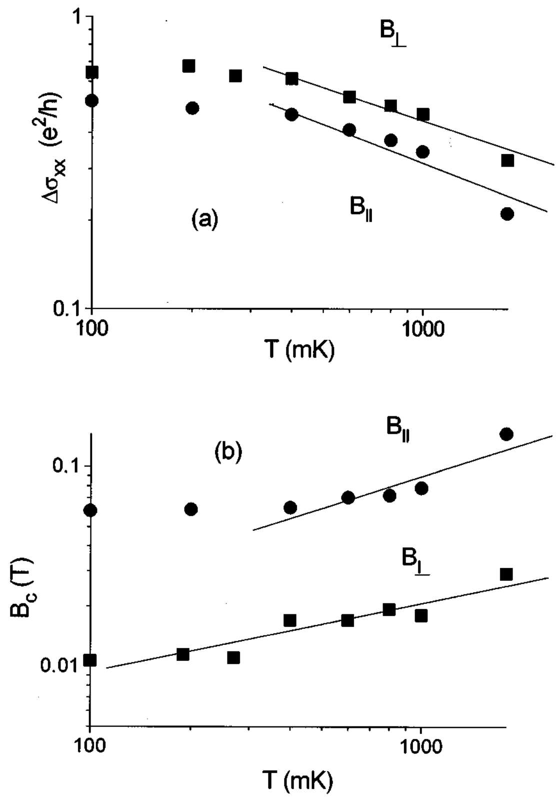

FIG. 3. (a) The rms amplitudes of the conductance fluctuations and (b) the correlation magnetic field, as a function of temperature, for a mesoscopic dimpled sample with periodicity $d = 0.3\ \mu$m, in perpendicular and parallel external magnetic fields. The straight lines are guides for the eye.

As expected for the flux cancellation effect, the correlation magnetic field $B_{c\parallel}$ in a parallel field is larger than $B_{c\perp}$ in a perpendicular field (Fig. 4). Moreover, the rms amplitude of the fluctuations in a parallel magnetic field is 1.5–2 times smaller than in a perpendicular field. The impurity configuration was changed by numerous thermocyclings up to room temperature, and by illuminating the sample at low temperature using an infrared light-emitting diode, in order to avoid sample-specific properties. To avoid any suppression of the rms amplitude by the magnetic field due to the influence on the orbital motion or Zeeman splitting, the fluctuations were measured within a similar magnetic-field range, up to 2 T. The effects on the conductance fluctuations by the random and inhomogeneous magnetic fields, respectively, were studied by measuring the magnetoresistance for the different angles between the field and the normal of the substrate plane (see Fig. 4). B_c starts to increase for angles larger than 50°–60°. Since the UCF's are exclusively caused by the enclosed flux through the electron trajectories lying in the substrate plane,[16] B_c should follow a $\cos^{-1}$ law for the conventional 2DEG. This also means that the Zeeman splitting does not play any role (up to 1.5 T). The correlation magnetic field in a parallel external field or a random effective magnetic field can be calculated from the random-walk model considered above. This gives $B_{c\parallel} = \phi_0/L_\varphi(d/S) =$

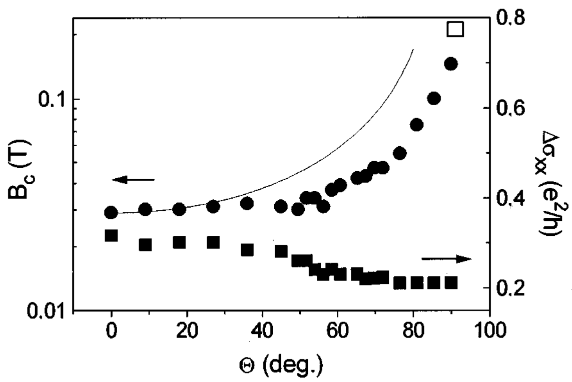

FIG. 4. The angular dependence of the correlation magnetic field (left axis), and the rms amplitude of the conductance fluctuations (right axis), for a mesoscopic dimpled sample with periodicity $d=0.3$ μm. The full line represents a $\cos^{-1}$ angular dependence of the rms amplitude, whereas $\square$ is calculated from $B_{c\parallel}/B_{c\perp}=L_\varphi/b$ (see text).

$2\phi_0 b/L_\varphi$, where b is the height of the dimples. The ratio between correlation fields for perpendicular and parallel fields is then given by $B_{c\parallel}/B_{c\perp}=L_\varphi/b$. In Fig. 4, $B_{c\parallel}$ calculated from this ratio is marked by the square. Considering that no adjustable parameters were used (b was taken from the SEM picture), the agreement is remarkable. For comparison, a curve following a $\cos^{-1}$ law is also shown in the figure. Figure 4 also shows the behavior of the rms amplitude of the fluctuations with rotation of the external magnetic field. A decrease of the amplitude starts at 40°–50°, and for a parallel field it is 1.4–1.5 times smaller. Some effects that may be responsible for the reduction of the amplitude in a magnetic field have previously been considered.[14] In the presence of a magnetic field $B>B_c$ the cooperon contribution to UCF's may be suppressed, reducing the rms amplitude by a factor of 2. Similarly, the lifting of the spin degeneracy also leads to a halving of the rms amplitude. However, neither effects can explain the observed decrease in the UCF amplitude. The suppression of the cooperon corrections occurs in small fields for both field directions. Furthermore, the magnitude of the characteristic field B_c would be expected to be larger for a parallel field than for a perpendicular field (in the same way as for the weak localization), so that if a difference were seen due to the cooperon suppression, the UCF's for the parallel direction would be larger, not smaller, as is experimentally seen. To lift the spin degeneracy one has to assume an unrealistically large anisotropy of the g factor, so that the Zeeman splitting would be much larger in a parallel field. Instead, one may speculate that in a sufficiently strong parallel magnetic field, the sign-alternating field can be considered to act as random magnetic scatterers. For the random magnetic scattering mechanism, the rms amplitude is found to be smaller than for the impurity scattering,[12] $\simeq 0.3e^2/h$. Further studies are obviously needed to understand the emergence of the magnetic scattering mechanisms in a random magnetic field.

In conclusion, we have studied negative magnetoresistance and universal conductance fluctuations in a 2DEG under inhomogeneous and random magnetic fields. Higher-order corrections to the flux through the closed electron trajectories govern the quantum interference in a random magnetic field, and determine the behavior of the negative magnetoresistance and UCF's. The higher-order effects may be of importance for the evaluation of e-e interactions in composite fermion systems.[17] However, the rms amplitude of the UCF's decreases when the field is changed from a slightly inhomogeneous to a random magnetic field. As a possible explanation we propose that the character of the electron scattering changes in this case, so that in a parallel field magnetic scattering becomes important in comparison with the conventional impurity scattering.

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