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Geometrical resonance in the resistivity of wide quantum wells

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Abstract

We studied size-effects in wide parabolic quantum wells by measuring the low field magnetoresistance in perpendicular and parallel magnetic field configurations. We have also studied the evolutions of the resistivity oscillations as a function of the electron sheet density N_S, by in situ illumination of the samples at the temperature of 1.5 K. We observed the maximum in the resistivity in the perpendicular magnetic field when the cyclotron radius was equal to the width of the electronic slab $R_c = W_e$. It allows to determine the width W_e as a function of the electron density. The oscillation amplitude decreases with N_S, which can be attributed to the increase of the probability of electrons to be specularly reflected by the boundary with increase of the electron slab width. In parallel magnetic field we also found oscillations corresponding to $R_c \approx W_e$ and $R_c/W_e \approx 2.2$ in agreement with quantum-mechanical calculations of the size effects in narrow quantum wires.

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Keywords: Parabolic quantum well; Boundary-roughness scattering; Three-dimensional electron gas; Size-effects

1. Introduction

The electrical resistivity of thin metallic films in the presence of magnetic field is strongly influenced by geometrical factors, especially when the electron mean free path l is comparable with the specimen dimensions. Experimentally, two types of configurations are well documented in literature, the first refers to magnetoresistance measurements, performed with the magnetic field vector transverse to the plane of the sample and also to the current (Sondheimer configuration) [1]. The second type of configuration corresponds to the case of measurements performed with the magnetic field directed along the plane of the film surface (MacDonald–Sarginson configuration) [2]. According to the work of Sondheimer, the resistivity of the film placed in perpendicular magnetic field reveals oscillations with successive maxima occurring for $R_c = nW$, where R_c is the cyclotron radius, W is the film thickness, and $n = 1, 7, 13, 19, \dots$. For the case of films placed in parallel magnetic field, a single peak is found at a position corresponding to $R_c/W \approx 1.8$ [2]. Thin films, in the presence of in-plane magnetic field, resemble narrow quantum wires in perpendicular field, in the sense that both have larger mean free path than the smallest dimension of the sample, therefore, similar features have also been found in the magnetoresistance of wires [3]. Such galvanomagnetic phenomena in thin metal films and also in quantum wires have been attributed to the combined influence of the magnetic field and surface roughness scattering [4].

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Parabolic GaAs/AlGaAs quantum wells contain the quasi-three-dimensional electron gas with high mobility and low electron density. The well is characterized by three parameters: the height of parabola $\Delta_1 \cong 200$ meV, the width of the parabola $W = 4000$ Å, and the height of the AlGaAs barrier $\Delta_2 \cong 22$ meV. The parabolic variations mimic the potential of a uniform positive three-dimensional charge n^+, which is proportional to the curvature of the grown parabolic potential, which is approximately given by $n^+ = 2\Delta_1\varepsilon/\pi e^2 W^2$ [5,6], where ε is the static dielectric constant and e is the electron charge. Electrons from the doped barriers will attempt to screen this pseudo-charge until equilibrium condition is reached. The width of the electronic slab W_e depends on the electron density into the well $W_e = N_S/n^+$. ($W_e \cong W$ when the well is full). The small Fermi energy of these systems, in comparison with metals, allows to study electron–electron correlation in three-dimensional systems. The selective doping of these heterostructures highly reduces the ionized-impurity scattering creating an almost ideal medium with low disorder to study size-effects in these systems. Strong manifestations are expected, because of the larger mean free path of the electrons, which can produce geometrical resonance by the interaction of the carriers with the boundaries of the samples before suffering internal collisions.

2. Results and discussion

In this work we report the observation of size-effects in the low field longitudinal magnetoresistance (R_{xx}) of very wide GaAs/AlGaAs parabolic quantum wells (PQW), with high mobility and low electron density, placed in perpendicular ($\Theta = 90^\circ$), tilted ($0^\circ < \Theta < 90^\circ$), and parallel magnetic field ($\Theta = 0^\circ$), configurations. We also performed measurements, for each of the previous cases, with a systematic variation of the electron sheet density N_S, by in situ illumination of the samples at the temperature of 1.5 K. For our study, GaAs/Al$_x$Ga$_{1-x}$As parabolic quantum well samples with 2000, 4000, and 6000 Å were grown by molecular beam epitaxy. The electron densities N_S in the dark are 3.9×10^{11}, 1.69×10^{11}, and 2.9×10^{11} cm^{-2} for the samples with 2000, 4000, and 6000 Å, respectively. After growth, substrate with PQW was processed into four terminal Hall bars. The distance between voltage probes was 100 μm, and the width of each bar was 50 μm. The measurements were performed using a superconductor magnetic system, with an AC current not larger than 10^{-6} A. For detection we used a lock-in technique and a chart recorder system.

For the case of the measurements performed with the magnetic field transversal to the surface of the sample and also to the electric current, stronger manifestations of size effects were observed in the case of the 4000 Å PQW. For this sample, the three dimensional pseudo-charge n^+ is $7,1\times10^{15}$ cm^{-3}. Under illumination, the electron sheet density N_S into the well is $\sim1.69\times10^{11}$ cm^{-2}, and the electron mobility is $\mu_e = 1.08\times10^4$ cm^2/V s. By effect of a step illumination, the electron concentration is steadily increased, by persistent photo-conductivity, until the PQW is full. In this case, the electron mobility is $\mu_e = 1.2\times10^4$ cm^2/V s, and the sheet concentration into the well is $N_S = 3.0\times10^{11}$ cm^{-2}, which corresponds to an effective electron slab width W_e of $\sim$4000 Å. In Fig. 1, we show the results of magnetoresistance measurements for different electron densities in perpendicular magnetic field. We observe the maximum in the resistivity in perpendicular magnetic field when the cyclotron radius is equal to the width of the electronic slab $R_c = W_e$, for a better presentation of the results, all curves were normalized in relation to the longitudinal resistance in zero magnetic field. The curve on the upper part of the figure corresponds to the electron concentration under illumination, each of the next curves corresponds to a one step of illumination. The curve on the lower part corresponds to the case when the well is full.

From the figure we observe that the oscillation amplitude decreases with the increment of N_S, it allows to determine the width W_e as a function of the electron density, which is shown in the inset of Fig. 1. This effect can be attributed to the increase of the probability of specular reflection of the electrons, by the boundaries of the sample when the electron slab width becomes wider. In order to prove these arguments, we performed numerical calculation of galvanomagnetic oscillations based on Sondheimer theory. According to Sondheimer when a thin film is placed in a configuration with the magnetic field applied in a direction which is perpendicular to the film plane, and

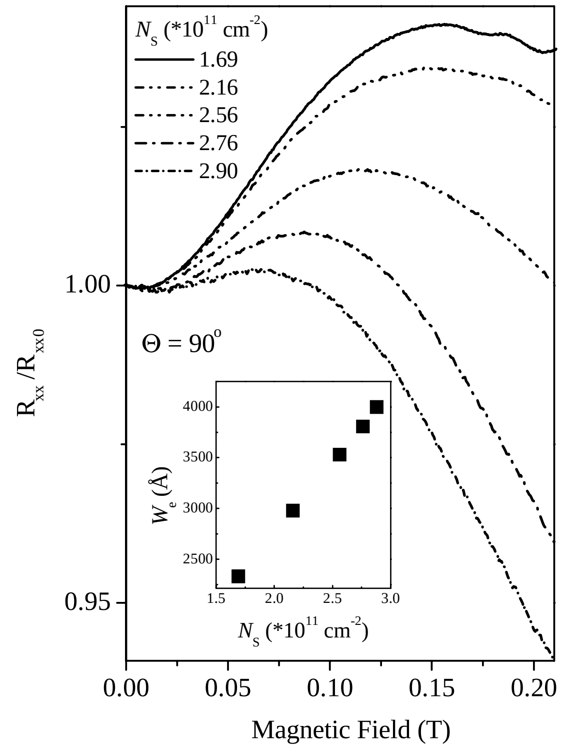

Fig. 1. Low field magnetoresistance for the 4000 Å PQW, taken at 1.5 K, for different electron concentrations (perpendicular magnetic field configuration). Inset, effective electron slab width vs. N_S.

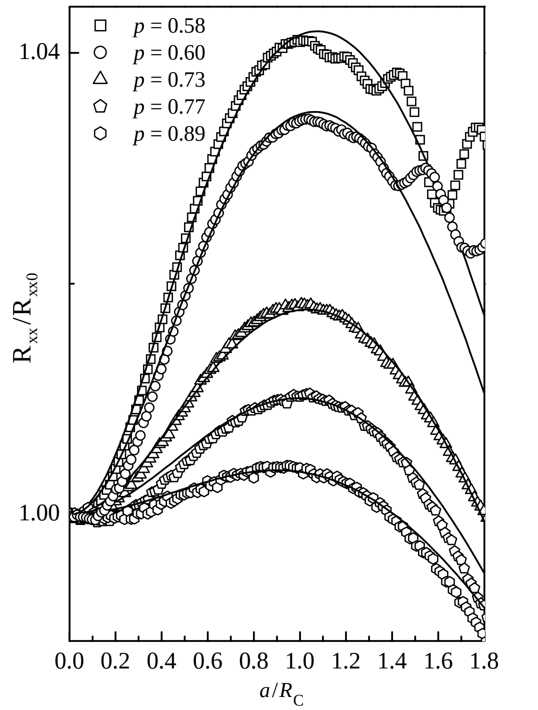

Fig. 2. Comparison between the normalized low field magnetoresistance of Fig. 1, plotted as a function of the ratio of the well width a, to the cyclotron radius R_c, and the calculated resistivity obtained for the application of Sondheimer theory (perpendicular field configuration).

to the direction of the electric current, the ratio of the resistivity $1/\sigma$ to that of the bulk sample $1/\sigma_0$ is given by [1]

$$\frac{\sigma_0}{\sigma} = \frac{\mathrm{Re}\{\phi(s)\}}{k} \tag{1}$$

with

$$\frac{1}{\phi(s)} = \frac{1}{s} - \frac{3}{2s^2}(1-p) \times \int_1^{\infty} \left(\frac{1}{t^3} - \frac{1}{t^5}\right) \frac{1-\mathrm{e}^{-st}}{1-p\mathrm{e}^{-st}}\,\mathrm{d}t \tag{2}$$

and $s \equiv k + \mathrm{i}\beta$, $k \equiv a/l$, and $\beta \equiv a/R_c$, where $\mathrm{Re}\{\phi_s\}$ stands for the real part of ϕ_s, a is the film thickness, l is the mean free path, R_c is the classical cyclotron radius, and p is the specular parameter which is zero for the case of total diffuse scattering. For our calculation we substitute the parameter a by the effective electron slab width W_e.

Fig. 2, shows the comparison between the normalized low field magnetoresistance curves of Fig. 1, and the results of the calculations performed with the aid of the Sondheimer model. From the experimental data of Fig. 1 we obtained the Fermi energy and the transport mean free path l for each stage of illumination; this allow us to obtain the experimental values of the parameter κ. With these experimental values of κ, we calculated the resistivity for each electron concentration using p as a parameter and the results are shown in Table 1. From our results, we observe an increase on the probability of the electrons to suffer specular reflection, with the boundaries of the parabolic well, when the well is being filled. For $N_S = 1.69\times10^{11}$ cm^{-2}, we have 42% of diffusive scattering, as the electron slab width increases, this amount falls until reaching a value of only 10% when the well is full. Self-consistent calculations, performed, for our samples, show that when the well is full, the electron slab has an almost

Table 1
Parameters obtained from the experimental measurements and specular parameter p obtained by fitting

$N_S \times 10^{11}$ (cm^{-2})	W_e (Å)	l (10^{-7} m)	κ exp.	p
1.69	2330	7.2	0.32	0.58
2.16	2979	7.4	0.40	0.50
2.56	3531	8.1	0.43	0.63
2.76	3807	9.8	0.40	0.77
2.90	4000	12.7	0.31	0.89

uniform distribution across the whole thickness of the well. This condition together with the low probability of electrons suffering internal collisions with ionized impurities, greatly increases the amount of specular back-scattering, this shows that the boundary scattering in wide AlGaAs/GaAs quantum wells has a predominant specular character due to the conservation of the electron longitudinal momentum, even, after several collisions with the sample edges. This is also in agreement with recent experiments performed in Ref. [7].

We have also performed magnetoresistance measurements for the case of magnetic field applied in the direction of the sample surface. In order to do this we varied, gradually, the in-plane component of the magnetic field, from perpendicular to parallel configurations. Fig. 3 shows the results of these measurements, for the case of $N_S = 2.16\times10^{11}$ cm^{-2}, as was shown in Fig. 1, there is a single resonance peak located at $R_c/W_e = 1$ in the perpendicular magnetic field. When we begin to increase, gradually, the in-plane component of the field, we observe a splitting of the single resonance peak in two peaks when $\Theta \approx 45^\circ$, however, the position of the first peak is maintained. Hereafter, for lower values of Θ, the peaks do not shift anymore and when the sample is in parallel magnetic field the two peaks are located at $R_c/W_e = 1$, and $R_c/W_e \cong 2.2$. These two peaks were also observed in the 2000 Å [8], and 6000 Å PQW samples placed, also, in parallel field.

For parallel configuration classical theory, based on classical electron trajectories and diffuse scattering at the surface of thin films, it was possible to reproduce only single magnetoresistance peak at $R_c/W \approx 1.8$. Quantum-mechanical calculations have only been reported for the resistivity of films only in the absence

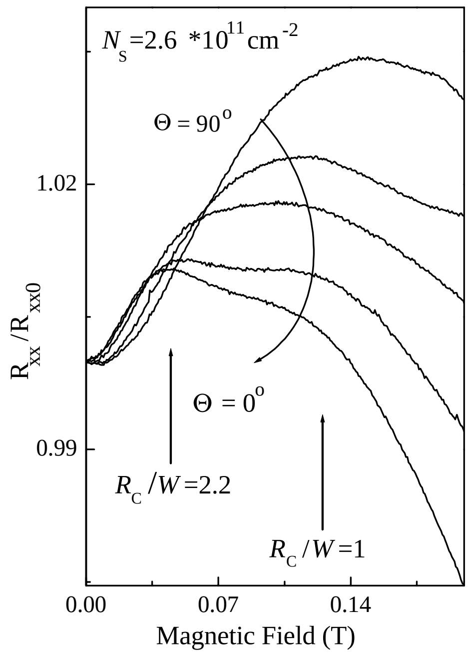

Fig. 3. Evolution of the anomalous magnetoresistance peaks, in tilted magnetic field at 1.5 K, for the 4000 Å PWQ sample. For this case the electron concentration is 2.6×10^{11} cm^{-2}.

of magnetic field or in perpendicular B. However, as mentioned at the introduction, the metal films in parallel field resemble narrow wires in a perpendicular field, therefore, we can use the recent results obtained for resistivity of quantum wires [4]. The essential ingredient of the quantum mechanical model is also the boundary roughness scattering. For wires, with effective width $W/\lambda_F = 3$ (λ_F: Fermi wavelength), the calculations reproduce the single magnetoresistance peak at $R_c/W \approx 1.8$, observed in narrow wires with diffuse wall scattering [3]. Moreover, for wider wires with width $W/\lambda_F = 5.5$, the additional peak at $R_c/W \approx 0.9$ has been predicted, which is not seen clearly in experiment [3]. This second peak is probably more sensitive to the shape of the confining potential and tends to be smeared out when confinement becomes softer. In parabolic quantum well in parallel field, we are able to see both peaks predicted for quantum wires. However, the exact position of peaks correspond to $R_c/W_e \approx 2.2$ and 1, which is 10–20% smaller than predicted for quantum wires. This discrepancy is not clear for us, and demands further quantum mechanical calculations of the resistivity of films in the presence of the in-plane magnetic field.

In summary, we have studied the anomalous low-field magnetoresistance in wide PQW. From the experimental data, for perpendicular configuration, we show the specular character of the boundary scattering in these systems through a comparison with a simple analytical model for magnetoresistance in thin films. For parallel configuration we report the observation of two peaks, located at $R_c/W_e = 1$ and 2.2, which is in accordance with quantum mechanical calculations of the resistivity of wires with boundary roughness scattering.

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References

[1] E.H. Sondheimer, Phys. Rev. 80 (1950) 401.
[2] D.K. MacDonald, K. Sarginson, Proc. Roy. Soc. 203A (1950) 223.
[3] T.J. Thornton, M.L. Roukes, A. Schrere, B.P. Van de Gaag, Phys. Rev. Lett. 63 (1989) 2128.
[4] H. Akera, T. Ando, Phys. Rev. B 43 (1991) 11 676.
[5] M. Shayegan, T. Sojoto, M. Santos, C. Silvestre, Appl. Phys. Lett. 53 (1988) 791.
[6] A.J. Rimberg, R.M. Westervelt, Phys. Rev. B 40 (1989) 3970.
[7] H. van Houten, C.W.J. Beenakker, J.G. Williamson, M.E.I. Broekaart, P.H.M. van Loosdrecht, B.J. van Wees, J.E. Mooij, C.T. Foxon, J.J. Harris, Phys. Rev. B 39 (1989) 8556.
[8] N.M. Sotomayor Choque, G.M. Gusev, J.R. Leite, A.A. Bykov, L.V. Litvin, N.T. Moshegov, A.I. Toropov, Physica B 298 (2001) 302.