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Magnetooscillations of electrons in nonparabolic confining potential

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Abstract

Shubnikov–de Haas (SdH) oscillations induced by the small additional perpendicular field are investigated in overfilled parabolic quantum well in the presence of the parallel magnetic field. The dispersion relation of the in-plane electron motion has a double minimum in contrast to the harmonic-oscillator model. Landau levels belonging to the additional subband minima are responsible for the observed behaviour of the SdH oscillations. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The parabolic confining potential is realized and investigated in many semiconductor low-dimensional devices such as quantum wells, wires and quantum dots. Theoretically, parabolic potential has attracted very much attention, since the energy spectrum can be calculated exactly within the harmonic-oscillator model. One of the example is the energy spectrum of the two-dimensional electrons, which are confined in the z direction by the parabolic potential in the presence of the in-plane magnetic field, and energy levels of the quasi-one-dimensional electrons in quantum wires subjected to perpendicular B. In this case the energy spacing can be solved explicitly, and each level is the hybrid electric-Landau subband [1]. Magnetoresistance reveals oscillations with B, which are due to the depopulation of subbands by the magnetic field and therefore, they resemble the conventional Shubnikov–de Haas oscillations. Depopulation of the subbands in a wide parabolic quantum well under in-plane magnetic field has been studied in Ref. [2]. Depopulation of levels in quantum wire in perpendicular magnetic field has been observed in Ref. [3]. However, we should note that an overfilled parabolic well should resemble a partially filled square well, and therefore harmonic-oscillator model is not valid more for realistic wide parabolic well. The calcula-

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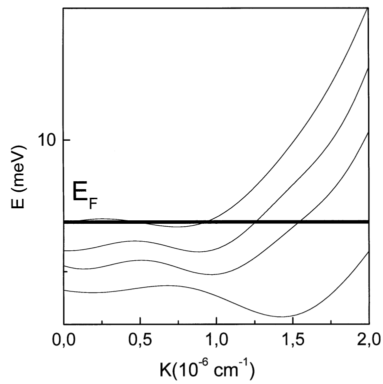

Fig. 1. Schematic representation of the subband energies of a parabolic quantum well as a function of the Femi vector in the presence of the in-plane magnetic field.

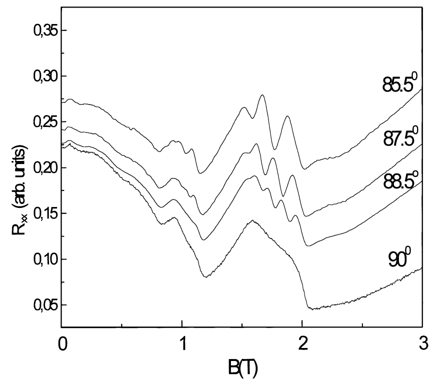

Fig. 2. Magnetoresistivity traces in parallel and tilted magnetic field.

tion of the fully self-consistent electronic structure of a overfilled parabolic quantum well and symmetric and asymmetric square quantum wells in the presence of the in-plane magnetic field has been done in Refs. [4,5]. It has been shown that the electron subband possesses the two local minima at finite Fermi vector on the dispersion curve. Fig. 1 shows schematically subband energies of a parabolic quantum well as a function of Fermi vector in the presence of the in-plane magnetic field. So when the Fermi level passes through this subband bottom one expects to observe two peaks in the magnetoresistance instead of the single peak for the pure parabolic confining potential case. Anomalous dispersion law with two local minima can be analyzed from Shubnikov–de Haas oscillations induced by sufficiently small additional perpendicular field. For ordinary parabolic dispersion law the electronic states become a series of discrete Landau levels (LL) of energy $E_N = h\omega_c(N + \frac{1}{2})$, with the cyclotron frequency ω_c given by eB/mc. For dispersion curves shown in Fig. 1 Landau levels demonstrate more complicated behavior. For example at definite interval of magnetic fields, levels with smaller quantum number N move downwards and intersect levels with larger N.

Here we report the measurements of the Shubnikov–de Haas oscillations in a quasi-parallel magnetic field. The behavior of the SdH oscillations cannot be explained in terms of a simple Landau fan chart. It is important to take into account additional local minima on the dispersion curve.

2. Experimental detail and results

The samples used are the GaAs-Al$_x$Ga$_{1-x}$As parabolic quantum well grown by molecular-beam epitaxy. After growth substrate with PQW was processed into Hall bar. Four-terminal resistance and Hall measurements were made down to 30 mK in magnetic fields up to 17 T. The measurements were performed with an AC current not exceeding 10^{-8} A. Resistance was measured for different angles between the field and substrate plane in magnetic field using an in situ rotation of the sample. The mobility of the electron gas in the well is $65 \times 10^3\ cm^2/Vs$, and concentration n_s in the dark is $3.9 \times 10^{11}\ cm^{-2}$. Three-dimensional pseudocharge n_{3D} is $2.1 \times 10^{16}\ cm^{-3}$ which corresponds the classical width of the 3D electron gas $w_e = n_s/n_{3D} = 190$ nm. This value is close to the geometrical width of the well, therefore the energy spectrum of a parabolic well can be roughly approximated by the spectrum of a square well.

Fig. 2 shows the magnetoresistivity traces in parallel and quasiparallel magnetic field. We see four peaks due to the depopulation of the electric subbands. Oscillations at strong magnetic fields split into two peaks, which we attribute to the two local minima of the subband structure. A small additional perpendicular field induces conventional Shubnikov–de Haas (SdH) oscillations. Surprisingly, the SdH oscillations are observed only on the top of the depopulation peaks in the interval of magnetic field, when Fermi level passes "pockets" in the dispersion law (Fig. 1). We did not find any SdH oscillations in stronger magnetic field. This observation disagrees with the conventional picture of the Shubnikov–de Haas oscillations, which requires an exponential growth of the oscillation amplitude with magnetic field. Assuming that from the $1/B$ periodicity of the anomalous SdH oscillations we can determine the electron density, we obtain $n_s = 1.3 \times 10^{12}\,\mathrm{cm}^{-2}$, which is 40 times larger, then the expected value for such angle $n_s/\cos\theta$, where n_s is the total electron density determined from the Hall measurements in the perpendicular field. To explain this value we should take the angle $\theta = 63^\circ$ instead of 88.5°. This difference is much larger than the precision of the angle measurements $\sim 1^\circ$.

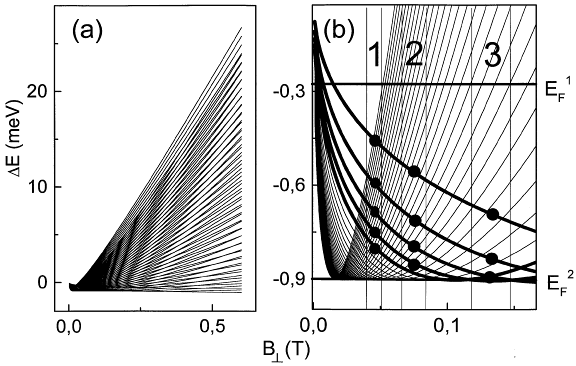

Fig. 3. (a) Landau levels as a function of the $B_\perp$. (b) Low field part of the Landau fan chart. Vertical lines indicate intervals of $B_\perp$, when SdH oscillations are observed for different tilt angles: (1) – 88.5°, (2) – 87.5°, (3) – 85.5°. Relative positions of the Fermi level during subband depopulation are indicated.

3. Theory and discussions

In the following, we calculate the energy spectrum of the two-dimensional electron in the wide square well subjected to an in-plane magnetic field. We take into account first-order perturbation theory. Treating the terms in Hamiltonian due to the in-plane magnetic field as a small perturbation we can obtain the first-order corrections to the energy of the each subband [6]:

$$E = \hbar^2(k_x - k_0)^2/2m + e^2 B_{\mathrm{II}}(\langle z^2\rangle - \langle z\rangle^2)/2m + \hbar^2 k_y^2/2m + E_n, \tag{1}$$

where $\langle ..\rangle$ denotes the quantum-mechanical expectation of length z and z^2 for the corresponding subband with energy E_n, and $k_0 = eB_{\mathrm{II}}\langle z\rangle/\hbar$. We can see from Eq. (1) that the symmetry $E(k_x) = E(-k_x)$ is broken for the asymmetric quantum well [5,6]. For the symmetric quantum well one should expect that $\langle z\rangle = 0$. However, it is necessary to consider self-consistent solution of the Poisson equation for the electrostatic potential and the Schrodinger equation for the wave function $\phi(k_x)$. In this case, the strong parallel magnetic field leads to the bending of the bottom of the square well, and, consequently, shifting of the wave function from the center of the well. The physical reason for the appearance of the nonzero $\langle z\rangle$ is the following. Consider electron moving parallel to the surface with velocity v_x. In the field directed along the surface it experiences a force $F = ev_x B_{\mathrm{II}}$, binding it to the surface. In the symmetric quantum well wave function $\phi(k_x)$ is shifted to the left side of the well, therefore $\langle z\rangle > 0$, and $\phi(-k_x)$ is shifted to another side, and $\langle z\rangle < 0$. In this case for the symmetric square quantum well:

$$E(k_x, k_y) = E(-k_x, k_y) = \hbar^2 k_x^2/2m - \alpha^{\pm} k_x + \hbar^2 k_y^2/2m + e^2 B_{\mathrm{II}}\langle z^2\rangle/2m + E_n \tag{2}$$

where $\alpha^+ > 0$ for $k_x > 0$, and $\alpha^- = -\alpha^+ < 0$ for $k_x < 0$. We also see that the effect of B_{II} is to make $E(k_x, k_y)$ anisotropic. We should note that for more realistic model $\langle z\rangle$ is not constant and increases with wave vector k_x, because the magnetic force increases with electron velocity, therefore, Eq. (2) is not valid at small k_x.

Now we discuss the effect of a small perpendicular field on the one hybrid subband. For simplicity, we consider isotropic energy dispersion. The energy levels in small perpendicular B are determined by the following equation [7]:

$$E_N = \hbar\omega_\mathrm{c}[N - (\gamma/2 + \gamma^2 N)^{1/2}], \tag{3}$$

where $\gamma = 2(\Delta/\hbar\omega_\mathrm{c})^{1/2}$, $\Delta = \alpha^2 m/2\hbar^2$ — energy minima in the center of the pocket of the dispersion curve (2). From the self-consistent calculations it is found that $\Delta \sim 1$–2 meV [4] at $B \sim 2$–3 T. Fig. 3(a) shows the dependence of the energy levels (3) on the perpendicular magnetic field. We see that at high magnetic field the dependence $E(B)$ resembles the conventional Landau fan chart, however at small field it is different. Low field dependence of the Landau levels is shown in Fig. 3(b). The levels at low field move downwards, pass minimum, and then move upwards. The Landau levels with small quantum number N move downwards at higher field, and therefore intersect LL with larger N, which already pass the minimum on the dispersion curve. We believe that this behavior can explain quantitatively our results. We cannot vary parallel and perpendicular components of the magnetic field independently. Parallel component of the magnetic field leads to the rise of all the subband levels due to the diamagnetic energy. Fig. 3(b) shows the relative position of the Fermi level during this diamagnetic shift. At the same time perpendicular component of the magnetic field increases too. Vertical lines in Fig. 3(b) shows the intervals of $B_{\perp}$, when the SdH oscillations appear in quasi-parallel field. Fermi level intersects LL which are moving downwards during the depopulation of the subband (the points of these intersections are marked by the circles in Fig. 3(b)). Landau levels moving upwards are not resolved due to the broadening and can be seen as a background with constant density of states. In experiments (Fig. 2) we find five oscillations in interval 1, 4 in 2 and 3 in interval 3, which is roughly consistent with the number of circles in Fig. 3(b). At larger tilt angles the levels moving upward become well resolved, and we start to observe SdH oscillations in all interval of B.

In conclusion, we have observed anomalous magnetooscillations in quasi-parallel B, which appear in the narrow interval of magnetic field. Effect is due to the Landau quantization of the states in the additional local minima of the dispersion curve.

Acknowledgements

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