Magneto-oscillations in a two-dimensional electron gas with a Penrose lattice of artificial scatterers

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Magnetoresistance of a two-dimensional electron gas in a quasiperiodic lattice (Penrose tiling) of antidots has been studied. Magnetoresistance oscillations in a magnetic field were found when the cyclotron diameter $2R_L$ was equal to the minimal distance between the centers of the antidots $d_{\text{min}}$, and for $2R_L=1.62d_{\text{min}}$. In contrast to the periodic lattice, where these oscillations originate from trajectories skipping along the array, in a quasiperiodic system commensurability oscillations of the magnetoresistance were suggested to be due to oscillations of the electrons scattered by the antidots.

Artificial arrays of scatterers fabricated by etching of holes with submicrometer diameters (antidots) through a high-mobility two-dimensional (2D) electron gas have attracted much attention because of the possibility of creating a lateral superlattice with different configurations of antidots: square,\textsuperscript{1,2} triangular,\textsuperscript{3} disordered.\textsuperscript{4} It has become possible to study the electronic properties for the transition from one type of scattering potential to another.

A lattice type with fairly interesting properties is a quasiperiodic lattice. For example, a one-dimensional Fibonacci-type superlattice exhibits a self-similar behavior: for an increase of the scale by a factor of $\tau^2$, where $\tau=(1+\sqrt{5})/2$ (a golden rule number), the superlattice transforms to itself. Experimentally, Ref. 6 demonstrates the point: correlation between photoluminescence spectra in magnetic field with changing of the cyclotron diameter by a factor $\tau^2$ has been observed. The two-dimensional version of a quasiperiodic lattice is a Penrose tile, the electronic properties of which, at zero magnetic field have been considered in Ref. 7. Electron lithography allows fabrication of samples with lateral dimensions less than 0.1–0.2 $\mu$m, but this size is still larger than the electron wavelength; as a consequence, the artificial superlattice does not change the energetic spectrum of electrons. However, the lateral superlattice essentially influences the electron’s dynamic and transport properties, in particular, when a magnetic field is present. Recently the Aharonov-Bohm effect and commensurability oscillations have been observed in a two-dimensional periodic array of antidots.\textsuperscript{1–3} In Refs. 8 and 9 the role of dynamic chaos in the transport properties of a 2D system with periodic arrays of scatterers has been demonstrated. In particular, it was shown that the stable electron trajectories skipping along arrays of antidots are responsible for commensurability oscillations in classically strong magnetic fields. These trajectories do not exist in quasiperiodic lattices. Therefore, the appearance of commensurability oscillations in a system with Penrose tiling is questionable. Another problem is the behavior of electrons in a weak or zero magnetic field. The scattering by antidots is dominant in comparison with scattering by impurities, and it is possible to investigate the dependence of mobility on the distance between antidots.

In this work we have fabricated a two-dimensional lattice of antidots of the Penrose-tiling-type, one of the modifications of a quasiperiodic lattice. For the fabrication of this lattice we used electron lithography and dry etching. The properties of the original heterostructures Al$_x$Ga$_{1-x}$As/GaAs were electron density $n_e=4\times10^{11}$ cm$^{-2}$ and the mobility $\mu=5\times10^5$ cm$^2$ V$^{-1}$ s$^{-1}$. The Penrose lattice is a certain variety of a two-dimensional quasiperiodic lattice. For its generation we used the Robinson construction described in Ref. 7. Robinson tiles can be obtained from two triangles $P$ and $Q$ which obey the recursion relation $P_n=2P_{n-1}+Q_{n-1}$, $Q_n=Q_{n-1}+P_{n-1}$. Basic triangles have angles $\pi/5$, $2\pi/5$, $2\pi/5$ and $3\pi/5$, $\pi/5$, $\pi/5$, respectively. The minimum size of a $P$ triangle, $d_{\text{min}}$, was 0.6, 0.8, and 1 $\mu$m for three different samples. Penrose tiling exhibits self-similar properties: with the increase of linear size $r^2$ times, a $P$ triangle obtained after $n$ iterations is similar to $P_{n-1}$ triangles, except for the lack of chirality. In the sites of a Penrose lattice, we patterned antidots with a diameter of 0.15–0.2 $\mu$m (see inset in Fig. 1). The antidot lattice covered the segment of the sample between the potential

![Graph](image)

**FIG. 1.** Magnetoresistance in patterned samples with different $d_{\text{min}}$: 1, 0.6 $\mu$m; 2, 0.8 $\mu$m; 3, 1 $\mu$m; $T=4.2$ K. Insets, mean free path as a function of $d_{\text{min}}$, fragment of antidots pattern.

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probes. Magnetoresistance was measured by the four-probe method, at a temperature of 1.3—4.2 K in magnetic fields up to 8 T.

Figure 1 shows the magnetic-field dependence of magnetoresistance for three samples with different values of $d_{\text{min}}$. It should be noted that we measured four samples, and for two samples with $d_{\text{min}} = 0.6 \mu m$, similar results were obtained. It can be seen that the resistance at zero magnetic field has different values for different $d_{\text{min}}$ and decreases with increasing $B$. In a strong magnetic field the difference between samples is not substantial. In a weak magnetic field, up to 0.4 T, additional oscillations are seen. The inset in Fig. 1 shows the electron mean free path $l$ as a function of $d_{\text{min}}$. It is apparent that this dependence is nonmonotonic: as $d_{\text{min}}$ grows, the decrease of length $l$ gives way to its increase. It should be noted that this anomalous decreasing of $l$ for $d_{\text{min}} = 0.8 \mu m$ is not due to the generation of additional defects, induced in the process of the reactive ion etching, because at $B = 1$ T the values for samples with $d_{\text{min}} = 0.6$ and $0.8 \mu m$ become comparable. Gusev et al. found that in a periodic lattice of scatterers the length $l$ is proportional to the distance between the antidots, i.e., $l = d - a$, where $d$ is the lattice period, $a$ is the diameter of the antidot; and it grows with the increase of the periodicity spacing. Electrons in a periodic array of scatterers are a system with dynamic chaos, in which randomization of electron trajectories occurs after several collisions with antidots. In Penrose tiling lattice the translation symmetry is violated, but short-range order is preserved; therefore it is important to take into account the dynamic chaos. The observed functional dependence of the mobility demands the theoretical consideration of the transport in the system.

In magnetic fields up to 10 mT we found negative magnetoresistance (nMR) due to the weak localization effects. In contrast to the periodic lattice for which, against the background of the nMR, there are some minima in field $B = \hbar c / 2 e b^2$, where $b = \sqrt{2(d - a / 2)}$; in a quasiperiodic system, $B$ dependence of nMR has no features and can be adequately described by theory using the coherence length $L_\phi$ as the adjustable parameter. In Ref. 3 the minima in negative magnetoresistance were associated with the Aharonov-Bohm effect for closed trajectories, when an electron travels ballistically from one antidot to another. Thus, for the observation of this effect in magnetoresistance only short order is necessary, which also exists in quasiperiodic systems. For example, we see in Fig. 1 small groups of similar antidot arrays for which it is possible to create trajectories with the same enclosed area. Absence of the feature associated with interference of stable trajectories in a quasiperiodic system could be due to the small number of these trajectories in comparison with the periodic lattice. We find a value for the coherence length $L_\phi = 0.4 - 0.5 \mu m$ at $T = 4.2$ K with weak dependence on $d_{\text{min}}$ and conductivity. This value is three times less than predicted by the theory, in the case of a conventional electron impurity system. Recently the same discrepancy between experiment and theory has been observed in periodic and disordered lattices of antidots.

We now discuss the behavior of magnetoresistance in magnetic fields up to 0.4 T. Figure 2 shows this dependence in more detail. One can see, that in samples with $d_{\text{min}} = 0.6, 0.8 \mu m$ three peaks are observed but in a sample with $d_{\text{min}} = 1 \mu m$ there are only two peaks. The positions of the peaks were found to be in agreement with commensurability requirements, consequently, $2R_L = d_{\text{min}}$, $\pi d_{\text{min}}$, $\pi^2 d_{\text{min}}$, where $\pi = 1.62$. It should be noted that the weak feature on the shoulder of the second oscillation with commensurability condition $2R_L = \pi d_{\text{min}}$ is also found for the sample with $d_{\text{min}} = 0.6 \mu m$. Thus, oscillations appear when the cyclotron diameter is equal to the size of the lattice units. The amplitude of oscillations decreases with increasing $d_{\text{min}}$ and decreasing magnetic field. This behavior is completely different from what is seen when commensurability oscillations are present in the samples with a periodic lattice. Next we compare the oscillations observed in samples with a Penrose lattice to commensurability oscillations perceived in periodic lattices with the same quality and ratio between the diameter of antidots and periodicity spacing. In a periodic system with $d = 0.6 \mu m$, two peaks in magnetoresistance were observed, when $2R_L = d$ and when $2R_L = 3.4d$. With the increase of $d$, the number of oscillation modes increases to 4—5. In the quasiperiodic system three strong peaks are seen for the sample with $d_{\text{min}} = 0.6 \mu m$ and only two peaks survive for the sample with $d_{\text{min}} = 1 \mu m$. The oscillation amplitude in the periodic lattice with $d = 0.6 \mu m$ at $2R_L = d$ equals 25% of the total resistance and increases to 50% for $d = 1 \mu m$. In quasiperiodic lattices the amplitude of oscillations for the same conditions, when $2R_L = d_{\text{min}}$ equals 12% of the total resistance, and decreases to 5.5% with an increase of $d_{\text{min}}$ to 1 µm. This is evidence that different mechanisms are responsible for oscillations in periodic and quasiperiodic systems. We know of two models which explain commensurability oscillations in a periodic lattice. The first model, constructed from the standpoint of dynamic chaos theory, gives evidence for the existence of stable trajectories running along the lattice array. In this case, the diffusion coefficient $D$ oscillates with the magnetic

![FIG. 2. Magnetoresistance of sample with antidots in low magnetic field up to 0.4 T with different $d_{\text{min}}$: 1, 0.6 µm; 2, 0.8 µm; 3, 1 µm; $T = 4.2$ K.](image-url)
field because of the contribution of both these trajectories, and the not quite stochastic trajectories, to the transport effects. For experimental conditions at magnetic fields $B > 0.2$ T, $\rho_{xx} \leq \rho_{xy}$ and $\rho_{xx} \sim \sigma_{xx}$; thus the maxima in $D$ are responsible for the maxima in the resistance. The other model was developed in Ref. 2. As was suggested in this work, the electron mean free path, and thus the diffusion coefficient for the scattered electrons, does not depend on $B$, but a fraction of these electrons $f_s$ has $B$-dependent oscillation modes. This suggestion contradicts the computer simulations of $D$, and therefore the commensurability oscillations of the resistance in periodic lattice was explained in Ref. 7 by the existence of the runaway electron trajectories. In our case of the quasiperiodic lattice, these trajectories cannot exist because of the lack of translation periodicity. Thus the coefficient $D$ does not depend on $B$. On the other hand, oscillations of $f_s$ with $B$ are determined by the short-range order and are not suppressed. Figure 3 shows typical trajectories of electrons which do not collide with antidots. The fraction for this orbit is $f_s = 1 - f_s$. We see that for Larmor radius $2R_L = d_{\min}$ and $2R_L = \pi d_{\min}$ these trajectories are simple circles around one antidot. For cyclotron diameter $2R_L = \pi d_{\min}$ and $2R_L = \pi d_{\min}$ the situation is more complicated. But we see in Fig. 3 that for Penrose tiling the group of 11 antidots is often repeated. In this case it could be possible that electron trajectories exist with diameter $\pi d_{\min}$ around this group, as shown in Fig. 3. The weak peak around $2R_L = \pi d_{\min}$ can be described by orbit around the six antidots. Thus, the oscillations in magnetoresistance are determined by the oscillations of $f_s$, as was considered in Ref. 2 for periodic lattices. The amplitude of the features in $f_s$ is small, in accord with the small amplitude of the resistance oscillations; this is in contrast with periodic lattices where oscillations of $D$ are dominant.\(^{1,3,8,9}\) The amplitude of the features in $f_s$ also decreases with increasing periodicity, i.e., decreasing of the ratio $d/a$, in agreement with experiment.

Thus from the measured magnetoresistance for 2D electrons in a quasiperiodic lattice we have pointed out two different mechanisms which are responsible for commensurability oscillations in the periodic lattice. Also, it should be noted that the quasiperiodic lattice has self-similar properties, which could be responsible for the dynamic chaos in the magnetic field, but further theoretical and experimental work is required to clarify this situation.

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