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# Commensurability oscillations in wide parabolic well in the presence of an in-plane magnetic field

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#### Abstract

The commensurability oscillations in antidot lattices with different periods are studied in the tilted magnetic field in wide parabolic quantum well with five occupied subbands. We find that the commensurability peaks are continuously transformed to new magnetoresistance oscillations, when magnetic field is directed parallel to the well plane. The positions of the new peaks correspond to  $R_c/W_e \approx 2.3$  and 1, where  $R_c$  is the cyclotron radius, and  $W_e$  is the quantum well width. This result is consistent with recent quantum-mechanical calculations of the magnetoresistance in the presence of boundary-roughness scattering. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Parabolic quantum well; Antidot lattice; Three-dimensional electron gas

## 1. Introduction

Magnetoresistance oscillations due to the size effects in thin metal films have been observed when a magnetic field is directed perpendicular to the plane of the film (Sondheimer effect [1]), and parallel to its plane and perpendicular to the current [2]. Because thin films, in the presence of the in-plane magnetic field, resemble narrow quantum wires in a perpendicular field, similar features have been found in the magnetoresistance of wires. The single low-field magnetoresistance peak has been found at a position corresponding to  $R_c/W \approx 1.8$ , where  $R_c$  is the cyclotron radius, and W is the wire width

[3]. Such size effect in thin metal films and quantum wires has been attributed to the combination of the influence of the magnetic field and surface roughness scattering. Quantum-mechanical calculations of the magnetoresistance in wires in the presence of boundary-roughness scattering have been reported in Ref. [4]. In addition to the peak at  $R_c/W \approx 1.8$  which was observed in the experiment [3], another peak at  $R_c/W \approx 0.9$  was also predicted for the wire width  $W/\lambda_F = 5.5$ , where  $\lambda_F$  is the electron Fermi wavelength. When the wire width and, consequently, the number of subbands decreases down to one, magnetoresistance peaks disappear, and the resistance decreases with field monotonically due to the formation of edge states.

Parabolic AlGaAs/GaAs quantum well allows one to study the evolution of a quasi-twodimensional towards a three-dimensional behavior.

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When the well is partially full, the electrons screen the parabolic potential and create a slab of comparable density over part of the width  $\lceil 5 \rceil$ . Therefore, when the well width increases, the thickness of the electron layer increases too, and two-dimensional gas is transformed to three dimensional. When the magnetic field is directed parallel to the well plane, magnetoresistance size effects, similar to the metal films and quantum wires are expected. In the present work we observe magnetoresistance oscillations in a 2000 Å width parabolic quantum well in the presence of the in-plane magnetic field. In order to prove that some of these oscillations in parallel magnetic field are due to the size effects, we measure commensurability effect in antidot lattice. In square antidot lattice, the maximum in the resistivity appears in the perpendicular magnetic field when the cyclotron diameter  $2R_{\rm c} = hk_{\rm F}/(\pi m_{\rm e}\omega_{\rm c})$ ,  $(\omega_{\rm c}$  is the cyclotron frequency, and  $m_{\rm e}$  is electron effective mass), is equal to the period of the array  $2R_c = d$ . We find that commensurability peaks are continuously transformed to new peaks at angles  $\Theta < 45^{\circ}$  ( $\Theta = 90^{\circ}$  refers to magnetic field perpendicular to the surface of the sample), which also exist in the presence of the in-plane magnetic field. We assume that the low-field oscillations in parallel magnetic field are associated with geometrical resonance when  $R_{\rm c} \approx W_{\rm e}$  and  $R_{\rm c}/W \approx 1.8$  in agreement with quantum-mechanical calculations of the size effects in narrow wires.

### 2. Results and discussion

The samples used are the GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As parabolic quantum well (PQW) grown by molecular beam epitaxy. On top of the semi-insulating substrate there is a 1000 nm GaAs buffer layer with 20 periods of AlAs(5 ML)GaAs(10 ML) superlattice, followed by 500 nm Al<sub>x</sub>Ga<sub>x-1</sub>As with x varying from 0.07 to 0.27, 100 nm Al<sub>0.3</sub>Ga<sub>0.7</sub>As with  $\delta$ -Si doping, Al<sub>0.3</sub>Ga<sub>0.7</sub>As undoped layer (spacer), the 200 nm wide parabolic well with composition variation 0 < x < 0.19. The electron density  $n_s$  in the dark is  $3.9 \times 10^{11}$  cm<sup>-2</sup>. After growth, substrate with PQW was processed into Hall bar, Fig. 1 shows schematically the conduction band edge in an empty (a) and partially full (b) parabolic

quantum well. Four terminal resistance and Hall measurements were made down to 1.5 K in a magnetic field up to 12 T. The distance between voltage probes was 250 µm, the width of the bar was 100 um. A lateral square superlattice (with periodicity 0.6, 0.7 and 0.8 µm) containing a macroscopic number of antidots  $(10^5)$  was fabricated in the center of the device between the potentiometric probes, using electron-beam lithography, to pattern the PMMA resist and plasma etching. The physical diameter of the antidots is  $\sim 0.1 \,\mu\text{m}$ . The measurements were performed with an AC current not exceeding  $10^{-6}$  Å. The resistance was measured for different angles between the field and substrate plane in magnetic field using an in situ rotation of the sample. Three-dimensional pseudocharge  $n_{3D}$  is  $2.1 \times 10^{16}$  cm<sup>-3</sup> which corresponds to the classical width of the 3D electron gas  $W_e = n_s/n_{3D} =$ 190 nm. For the wide parabolic well the sample layer is close to the geometrical width of the well, therefore the energy spectrum  $E_i$  of a parabolic well can be roughly approximated by the spectrum of a square well  $E_i = i^2 (h/W_e)^2 / 8m_e$ , where  $m_e$  is an effective electron mass. The mobility of the electron gas in the well is  $65 \times 10^3 \text{ cm}^2/\text{V} \text{ s}$ .

Fig. 1c shows the magnetoresistance oscillations for different angles  $\Theta$  between the field and the substrate plane. In a perpendicular magnetic field at  $\Theta = 90^{\circ}$  we see two commensurability peaks  $2R_{\rm c} = d$  and 2.5d in agreement with measurements in two-dimensional electron gas with single subband occupation [6,7]. Indeed in parabolic wells without antidot no commensurability peaks in perpendicular magnetic field are observed. Surprisingly, in a tilted magnetic field the commensurability peaks are continuously transformed to new peaks at higher magnetic field, and at tilt angle  $\Theta < 30^{\circ}$ peaks are not shifted anymore with angle and coincide with the magnetoresistance oscillations in parallel field which are also observed in samples without antidots [8] (see Fig. 2). Fig. 3 shows the dependence of the main commensurability peak position on the tilt angle. We see that the position of the main peak is not following the  $(\sin \Theta)^{-1}$  law as expected for two-dimensional electron gas.

At tilt angle  $\Theta < 45^{\circ}$  new peaks appear in an unpatterned sample with wide parabolic well, their positions are not moved with angle, and are finally



Fig. 1. Schematic illustration of the empty (a) and partially full (b) parabolic well. (c) Magnetoresistance commensurability oscillations in antidot lattice with period 0.7  $\mu$ m for different angles between the applied magnetic field and the substrate plane, T = 1.5 K.



Fig. 2. Magnetoresistance oscillations at low magnetic field and in tilted angles for the unpatterned region of the PQW sample.

coincident with peak positions in a sample with antidot lattice in parallel magnetic field. Condition  $2R_c = d$  gives us information about Fermi velocity and Fermi energy of the electrons that contribute to the conductivity peak. From comparison of the peak position in the samples with different periods we find  $E_F = 4 \text{ meV}$  which agrees with the three-dimensional Fermi energy of electrons in well.



Fig. 3. Position of the main commensurability peak as a function of the angle between the applied magnetic field and the normal to the substrate, triangles: position of the second low-field peak in unpatterned sample, T = 1.5 K. Thick line:  $(\sin \Theta)^{-1}$  dependence.

The self-consistent calculations of the energy levels in well give five occupied subbands. For independent two-dimensional subbands we have to observe several commensurability peaks. The strong scattering between subbands may result in a single main commensurability peak. The threedimensional character of the electron motion in wide PQW can also be responsible for anomalous oscillations in the presence of an in-plane magnetic field. However, classical theory based on classical electron trajectories and diffuse scattering at the surface of the film was able to reproduce only a single magnetoresistance peak at  $W/R_c \approx 0.55$ .

Quantum-mechanical calculations have been reported of the resistivity of films only in the absence of a magnetic field [9], or in perpendicular B [10]. However, as we mentioned above, the metal-films in parallel field resemble narrow wires in a perpendicular field, therefore we can use the recent results obtained for resistivity of the quantum wire  $\lceil 4 \rceil$ . The essential ingredient of the model is the boundary roughness scattering. In parabolic quantum well the roughness of the walls results from random fluctuations of the remote-ionized impurity concentration and the alloy composition. A quantum mechanical model of wires with boundary roughness and effective wire width  $W/\lambda_{\rm F} = 3$  reproduces the single magnetoresistance peak at  $W/R_c \approx 0.55$ , observed in narrow wires with diffuse wall scattering [3]. Moreover for wider wires with width  $W/\lambda_{\rm F} = 5.5$ , the additional peak at  $W/R_{\rm c} \approx 1.1$  has been predicted, which is not seen clearly in experiment [3]. This second peak is probably more sensitive to the shape of the confining potential and tends to be smeared out when confinement becomes softer. In parabolic quantum well in parallel field we are able to see both peaks, predicted for quantum wires. However, the exact position of peaks correspond to  $W_e/R_c \approx 0.45$  and 1, which is 10-20% smaller, than predicted for quantum wires. This discrepancy is not clear to us, and demands further quantum mechanical calculations of the resistivity of films in the presence of the in-plane magnetic field.

In summary, we have investigated the anomalous low-field magnetoresistance oscillations in wide parabolic quantum well in a parallel magnetic field. We attribute such oscillations to size effect due to commensurability between cyclotron radius and well width. To demonstrate it, we study the other type of commensurability effect-magnetoresistance oscillations in antidot lattice in a tilted magnetic field. We find that the commensurability peaks in a perpendicular field are continuously transformed to new magnetoresistance oscillations in parallel field with rotation of B. The further study of the size effects in wide semiconductor well will allow one to obtain information about surface roughness in such systems.

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