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Physica B 298 (2001) 306–309

**PHYSICA B**

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## Quantum Hall effect in a wide parabolic well

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### Abstract

We examine the activated conductivity for Landau levels at filling factors  $N = 1, 2$  in a parabolic  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  well down to 30 mK. We assume that the possible mechanism of transport is a hopping conductivity in the middle between overlapping levels. These results demonstrate that the plateau at  $N = 1, 2$  in quantum Hall effect are due to the energy gaps between electric subbands. © 2001 Elsevier Science B.V. All rights reserved.

**Keywords:** Quantum Hall effect; Parabolic well

### 1. Introduction

The quantum Hall effect (QHE) is characterized by the narrow resistivity peaks separated by deep minima (see for review, [1]). These resistivity minima arise due to the localization of the electron states at the Fermi energy, when it lies in the gaps between Landau or spin split levels. In this regime the resistivity is determined by the electron thermal activation with an energy gap  $\hbar\omega_c/2$  where  $\omega_c = eB/mc$ —cyclotron frequency for Landau levels and  $g\mu B/2$  for spin split levels (here  $\mu$  is the Bohr magneton,  $g$  is the effective factor Lande). The temperature dependence of the resistivity minima

can therefore be used to study energy gaps in the QHE. Number of experiments have been performed to study the exchange enhanced Landau and spin splitting [2], composite Fermions cyclotron energy [3] and skyrmions [4,5]. In conventional picture QHE are predicted to disappear for a sufficiently wide quantum well. Fig. 1 shows schematically the energy spectrum as a function of magnetic field  $B$  for a wide square quantum well. At sufficiently high  $B$ , the Fermi energy crosses energy levels corresponding to the lowest Landau level of each subbands. Therefore, it is expected that for wide quantum well last resistivity minima could be due to the energy gap between electric subbands, and not Landau levels. In real system the energy levels will have finite widths because of the disorder, therefore last resistivity minima disappear for a sufficiently thick layer, when corresponding electric subbands overlap. However, magnetoresis-

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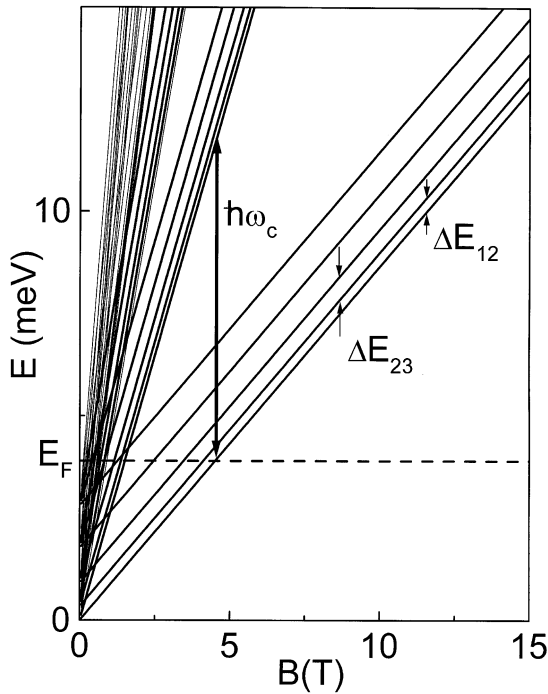


Fig. 1. Energy diagram for a wide quantum well. The cyclotron and subband energies are indicated. Dashed line – the Fermi energy at zero magnetic field.

tance should show oscillations as Fermi energy crosses degenerated Landau subbands. A three dimensional electron gas (3DEG) is the presence of a strong magnetic field is also expected to show interesting properties. It has been predicted a charge-density wave or a Wigner crystal at high magnetic field for a such system [6]. However in the doped three dimensional semiconductors the electron impurity interaction is very strong and can destroy these exotic ground states.

The system which can be used in order to study the evolution of states of a quasi-two-dimensional toward three dimensional behavior is a wide partially filled parabolic quantum well (PQW). Considerably larger mobility in comparison with conventional bulk 3DEG has been achieved by removing the dopant atoms from the quantum well [7]. Very recently the quantization of the Hall resistance and the appearance of minima in the diagonal resistance has been observed in a three dimensional disordered GaAs film [8]. It has been assumed that the Coulomb gap due to the electron

electron interactions occurs in high magnetic field, which leads to the deep minima in the dissipative and quantization of the diagonal components of the conductivity. Therefore further experimental and theoretical works are necessary to clarify the role of the electron–electron interaction on the single electron gaps (electric and Landau) in the wide quantum well.

## 2. Experimental results and discussion

The samples used are the GaAs–Al<sub>x</sub>Ga<sub>1–x</sub>As PQW grown by molecular-beam epitaxy. On the top of the semi-insulating substrate there is 1000 nm GaAs buffer layer with 20 periods of AlAs(5 ML)GaAs(10 ML) superlattice, followed by 500 nm Al<sub>x</sub>Ga<sub>1–x</sub>As with  $x$  varying from 0.07 to 0.27, 100 nm Al<sub>0.3</sub>Ga<sub>0.7</sub>As with  $\delta$ -Si doping, Al<sub>0.3</sub>Ga<sub>0.7</sub>As undoped layer (spacer), the 200 nm wide parabolic well with varying  $0 < x < 0.19$ . Two structures have been studied A and B, the main difference between these is that the parabolic well A has a spacer with smaller thickness 100 Å in comparison with 400 Å for sample B. It leads to the difference in the electron density: sample A has concentration  $n_s$  in the dark  $3.9 \times 10^{11} \text{ cm}^{-2}$ , sample B— $0.54 \times 10^{11} \text{ cm}^{-2}$ . After growth substrate with PQW was processed into Hall bar. Four-terminal resistance and Hall measurements were made down to 30 mK in magnetic field up to 17 T. The distance between voltage probes was 250  $\mu\text{m}$ , the width of the bar was 100  $\mu\text{m}$ . The measurements were performed with an AC not exceeding  $10^{-8} \text{ A}$ . Three dimensional pseudocharge  $n_{3D}$  is  $2.1 \times 10^{16} \text{ cm}^{-3}$  which corresponds the classical width of the 3D electron gas for the sample A  $w_e = n_s/n_{3D} = 190 \text{ nm}$  and  $w_e = 26 \text{ nm}$  for sample B. Therefore we can compare the behavior of the QHE for the narrow and for wide electron layers. For the wide parabolic well the sample layer is close to the geometrical width of the well, therefore the energy spectrum  $E_i$  of a parabolic well can be roughly approximated by spectrum of a square well  $E_i = i^2(h/w_e)^2/8m_e$ , where  $m_e$  is the effective electron mass. The mobility of the electron gas in the well is  $270 \times 10^3 \text{ cm}^2/\text{Vs}$  for sample B and  $65 \times 10^3 \text{ cm}^2/\text{Vs}$  for sample A.

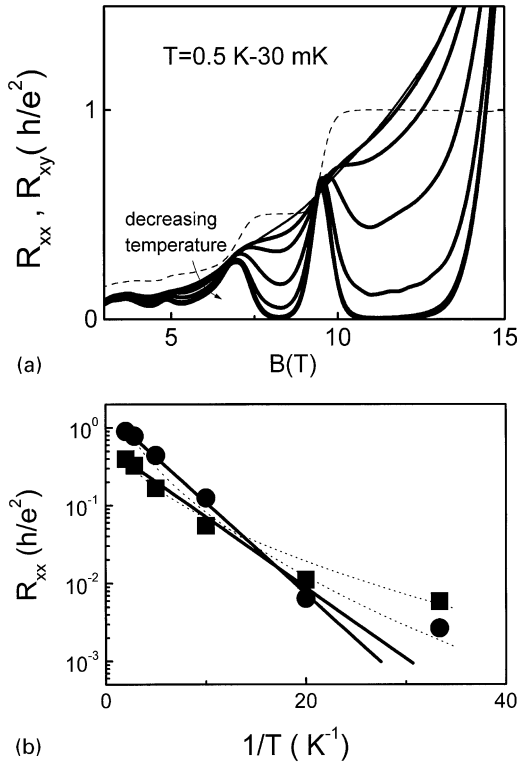


Fig. 2. (a) Longitudinal resistivity for different temperatures. Dashed curve corresponds to the Hall resistivity at  $T = 50$  mK. (b) Arrhenius plot of  $R_{xx}$  at  $N = 1$  and  $N = 2$  vs  $1/T$  (circles – resistance at  $N = 1$ , squares –  $N = 2$ ), dashed curves – Eq. (1).

The magnetoresistance and Hall effect data for sample A at different temperatures are shown in Fig. 2a. We see that in strong magnetic field and low temperatures the sample reveals the conventional QHE. However, the resistance peaks at filling factors  $N = \frac{3}{2}$  and  $\frac{5}{2}$  are completely smeared out at 0.5 K. For parabolic well with smaller layer width (sample B) QHE is still clearly seen at  $T = 0.9$  K. Fig. 2b shows activated resistivity at  $N = 1$  and 2 minima which can be fit to the Arrhenius plot. The energy gaps calculated from the Arrhenius plot are very small  $\sim 0.02$  meV. Some of the low-temperature experimental data in the QHE regime display hopping conductivity behavior [9]:

$$\rho_{xx} \propto \exp[-(T_0/T)^{1/2}], \quad (1)$$

where  $T_0 \approx e^2/\epsilon\xi$ ,  $\epsilon$  is the dielectric constant, and  $\xi$  is the electronic localization length. Fig. 2a shows

that our transport data also can be successfully fit to the formula (1). We don't find zero resistance at magnetic field  $B < 7$  T, however background magnetoresistance cannot be explained by hidden disordered electron layer, because we obtain zero resistance at stronger magnetic field.

We also find that at  $B > 1.2$  T all oscillations have a much stronger temperature dependence than oscillations at lower field. Peak at  $B = 5$  T (Fig. 2a) and neighbors minima are also completely smeared out at  $T = 0.5$  K. We should note that for the sample B the energy gaps calculated from the Arrhenius plot for minima 1 and 2 are coincident with the gaps which are expected for Landau and spin splitting. Numerical self-consistent calculations for parabolic well of width  $w = 200$  nm yield the following energies for the first 5 electric subbands:  $E_1 = 0.035$  meV,  $E_2 = 0.37$  meV,  $E_3 = 0.94$  meV,  $E_4 = 1.8$  meV,  $E_5 = 3$  meV, and Fermi energy  $E_F = 3.9$  meV. Within a single electron model, application of a magnetic field perpendicular to the plane of the quantum well forms a fan of Landau levels originating from the energy levels of each electric subband (Fig. 1). Thus, minima at  $N = 1, 2$  correspond to the Fermi energy lying in the energy gap  $\Delta E_{12}$  and  $\Delta E_{23}$ , where  $\Delta E_{ij}$  is the distance between  $i$  and  $j$  subbands at zero magnetic field. In the strong magnetic field only last Landau level of the lower electric subband is occupied, thus the last minimum should be observed at higher field. It is following from the fact, that in 2D case the position of the minima in a strong magnetic field do not reflect the separation of the levels but their degeneracy, and minimum occurs when  $n_s = hB/(Ne)$  (for spin resolved levels). From the Hall concentration at low field we find  $n_s = 3.9 \times 10^{11} \text{ cm}^{-2}$ , which gives the position of the last minimum  $B = 16.25$  T. Fig. 2a shows that at  $B > 13$  T the diagonal resistance starts to grow, however the well developed Hall plateau are still seen at higher magnetic field. This behavior corresponds to the metal-Hall insulator transition. Therefore for the last minimum in the diagonal resistance Fermi level does not lie in the center of the  $\Delta E_{12}$  gap. This shift of the metal Hall insulator transition to the lower field could be due to the smallness of the  $\Delta E_{12}$  energy gap.

We attribute the temperature dependence of the resistivity to the variable-range hopping in the

presence of the Coulomb gap [10]. The values of  $T_0$  obtained from the fitting of the experimental results to the Eq. (1) are 1 and 2.1 K consequently, for  $N = 1$  and 2 levels. Assuming that the localization radius  $\xi$  is of the order of the magnetic length  $l_B$  ( $\sim 100$  Å in this field), one may expect that  $T_0 \approx 132$  K. This temperature is much larger than what we found in experiment. The same discrepancy has been found for spin split levels in InGaAs/InP samples [11]. Such behavior can be explained by the divergence of the localization length, when two levels strongly overlap [10]. In this case

$$\xi \sim l_B(\Gamma/\Delta)^{2\gamma}, \quad (2)$$

where  $\gamma = 2.3$  is critical exponent in localization theory,  $\Delta$  is the energy gap between levels, and  $\Gamma$  is the broadening of the level. It is worth to note also, that in PQW electron should tunnel in  $z$  direction too, therefore  $l_B$  can be replaced by  $(l_B \times Z_{n,m})^{0.5}$ , where  $Z_{n,m}$  is the average wave function shift from level  $n$  to  $m$ . From comparison of the experimental results for  $T_0$  and Eqs. (1) and (2) we obtain the level broadening  $\Gamma \cong 0.7\text{--}1.2$  meV. Thus the energy gap is approximately two times smaller than the broadening of the levels. In this case the two peaks in resistivity are still resolved, and the hopping conductivity in the middle between them can be obtained. The same situation has been observed for the spin split level in InGaAs/InP heterostructures, where  $T_0$  was found to be 22 K [11]. In our case electric subband separation is much less than

spin splitting, and therefore  $T_0$  is found in order of 1 K. Small values of  $T_0$  support schematic energy diagram in wide PQW (Fig. 1) and conclusion, that the plateau in QHE are due to the gap between different electric subbands.

### Acknowledgements

The work is supported by CNPq, FAPESP and USP-COFECUB.

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