Spin-dependent Hall effect in a parabolic well with a quasi-three-dimensional electron gas

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We study the Hall effect in wide $Al_cGa_{c-1}As$ parabolic wells in the presence of the tilted magnetic field. The Hall resistance is described by equations $R_{xy}/\cos\Theta = -B/en_s$ at $B<4$ T, and $R_{xy}/\cos\Theta = -A\times(B-B_0)/en_s$ at $B>4$ T, where n_s is the electron density, $B_0=2-2.6$ T, A is the temperature dependent coefficient, and Θ is the angle between the magnetic field and the normal to the well plane. The effective g factor in such materials depends on the Al composition and changes the sign along the well width. In the presence of the strong tilted magnetic field electron moving along the z direction acquires a spin flip process, which is strongly suppressed at low temperatures, and leads to the change of the Hall effect slope.

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I. INTRODUCTION

The electron spin is a subject of intense current investigation in the area of spintronics—a new and very promising direction of research that may have important practical applications in microelectronics. Manipulation with the electron g factor, in particular its sign, is an effective method to vary the direction of the spin polarization of electrons, which is the key factor in spintronics.

A promising system providing effective control and manipulation with the electron spin is a remotely doped $Ga_{1-c}Al_c$ As parabolic quantum well,[1,2] because the spin properties of such materials depend strongly on the Al composition c. Assuming as [001] the growth direction, and taking as $z=0$ the position of the pure GaAs material, an effective harmonic potential is given by $U=m\Omega^2z^2/2$ with $\Omega = a(2/m)^{1/2}$ and effective mass m, when a composition profile $c(z)=az^2$ is achieved. The effective g factor changes with composition:[3] $g(c)=-g_0+g_1c$, where $g_0=0.44$, and $g_1=2.7$. Therefore, the g factor increases monotonically from $g=-0.44$ (middle of the well) to $g=+0.4$ at the edge of the well ($c=0.3$), and changes sign at $c=0.13$. Figure 1 shows the variation of the g factor in this parabolic well along the z axis. An application of the strong perpendicular magnetic field leads to the Zeeman splitting of the Landau levels, which is proportional to the average g factor in the parabolic well. Recently spin precession of two-dimensional (2D) electrons in a 1000 Å parabolic well in a perpendicular magnetic field has been measured from photoluminescence as a function of the gate voltage.[4] The electric field displaces the electron wave function along the z axis and leads to the strong variation of the average g factor, and consequently, variation of the spin lifetime. This ability to tune the local electron g factor allows us to fabricate, in principle, the spin valve transistor or other spintronic devices, however, the existence of such a spin-dependent property has not been studied yet in transport coefficients. Only the recently spin-related quantum Hall ferromagnets at Landau filling factor $\nu=2$ have been studied in wide (1000–3000 Å) parabolic wells from magnetotransport measurements.[5]

In the present work we study the Hall effect in wide parabolic wells in a quasiparallel magnetic field. For tilt angle $\Theta \lesssim 90$ deg and low temperature we found that the Hall resistance at $B>4$ T increases its slope twice and becomes temperature dependent. The ordinary Hall effect recovers at high ($T=50$ K) temperature and perpendicular magnetic field. We attribute the Hall slope change in our parabolic well to the unusual spin properties. In strong parallel magnetic field, when the cyclotron diameter becomes smaller than the well width, the spectrum turns on the spectrum of the three-dimensional (3D) gas[6] in the quantum limit with local g-factor variable along z. Moreover, the local effective g factor in such a structure changes sign along the well width. The ordinary Hall effect arises from the Lorentz force that acts on a moving charge. In the tilted magnetic field, an electron in the parabolic well moves in the y (perpendicular to the current flow) and z directions. However, the motion in the z direction requires a spin flip process, which is suppressed in

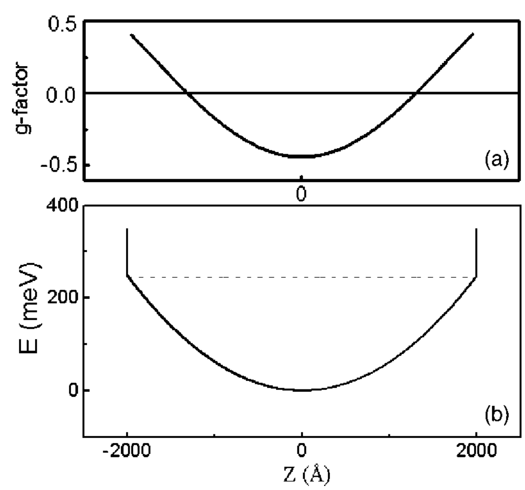

FIG. 1. The effective g factor (a) and variation of the band (b) along the z direction for a 4000 Å-wide parabolic quantum well with composition $c=0$ and $c=0.29$ at the center and the edge of the well, respectively.

TABLE I. The sample parameters.

Sample	Spacer (Å)	W (Å)	n^+ (10^{16} cm^{-3})	n_s (dark) (10^{11} cm^{-2})	W_{eff} (Å)	μ (dark) (cm^2/V s)	n_s (after illumination) (10^{11} cm^{-2})	W_{eff} (Å)	μ (after illumination) (cm^2/V s)
2537	250	500	47.6	4.4	92.4	590 000	6.4	133	440 000
2536	250	750	21	4.4	210	322 000	6.0	290	302 000
2577	200	1000	11.9	4.2	353	90 000	5.9	495	210 000
2579	300	1000	11.9	4	340	131 000	6.0	510	185 000
2580	400	1000	11.9	3.4	285	232 000	4.8	403	342 000
2496	200	1500	5.3	3.5	650	150 000	5.5	1040	220 000
2535	200	1700	4.1	3.2	784	220 000	5.0	1225	200 000
2534	150	2200	2.7	3.1	1150	186 000	5.0	1850	180 000
AG662	100	4000	0.88	1.5	1700	120 000	3.5	4000	240 000

the case of the small spin-orbit interaction. Since in the tilted field the electronic motion in the z direction and y direction are strongly coupled, the transversal component of the conductance is suppressed, and Hall resistance grows.

II. EXPERIMENTAL RESULTS

The samples were made from a $Ga_{1-c}Al_cAs$ parabolic quantum well grown by molecular-beam epitaxy. It included a 500, 750, 1000, 1500, 1700, 2200 and 4000 Å-wide parabolic $Ga_{1-c}Al_cAs$ well with the Al content varying between 0 and 0.29, bounded by undoped $Ga_{1-c}Al_cAs$ spacer layers with δ-Si doping on two sides.[6] The characteristic bulk density is given by equation $n_+ = \Omega_0^2 m^* \varepsilon / 4\pi e^2$. The effective thickness of the electronic slab can be obtained from the equation $W_{eff} = n_s / n_+$. For a partially filled quantum well, W_e is smaller than the geometrical width of the well W. The mobility of the electron gas in our samples was 590×10^3 cm^2/V s for narrow wells and $\sim 200 \times 10^3$ cm^2/V s for wider parabolic wells. We varied the electron sheet density by illumination with a red light-emitting diode. The summary of the sample parameters are shown in Table I. We see that before illumination our quantum wells were partially full with 2–5 subbands occupied. The ratio between the effective thickness of the electronic slab and geometric width $f = W_{eff}/W$ was less than 0.3 for $W < 1000$ Å parabolic wells. After illumination the effective thickness increases, however, the electron density is saturated, and the quantum well still remains partially occupied (see Table I). We are able to obtain the ratio $f=1$ only for a 4000 Å wide sample. The energy spectrum of the full PQW is more similar to the square quantum well than to a harmonic potential.

The test samples were Hall bars with the distance between the voltage probes $L = 200$ μm and the width of the bar $d = 100$ μm. Four-terminal resistance R_{xx} and Hall R_{xy} measurements were made down to 1.5 K in a magnetic field up to 12 T. The current was directed along the Hall bar (x axis). We rotate sample *in situ*, so that the magnetic field could be tilted with respect to the sample $(x-y)$ plane in x or y directions. We denote the angle between B and the normal to the sample plane by Θ.

Figure 2 shows longitudinal R_{xx} and Hall R_{xy} resistances of 4000 Å-wide parabolic $Ga_{1-c}Al_cAs$ well at $\Theta \approx 89$ deg as a function of the applied magnetic field for different temperatures. The sample was illuminated in order to approach wider width of the electronic slab. The magnetic field is directed along the current flow. We may see that the magnetoresistance reveals oscillations, sometimes called diamagnetic Shubnikov de-Haas (SdH) oscillations, which result from the combined effect of the electric and magnetic fields. In a quantum well with several subbands occupied, such oscillations are interpreted as the magnetic depopulation of the two-dimensional levels. Figure 2 shows that R_{xx} exhibits 5 oscillations, therefore 5 subbands are depopulated in this 4000 Å-wide parabolic well. When the temperature increases, the oscillations are smeared out, and only the minimum corresponding to the depopulation of the last Landau level is seen. The Hall resistance demonstrates linear dependence on the magnetic field at low B, and at a higher field, R_{xy} deviates from the former linear dependence, and its slope

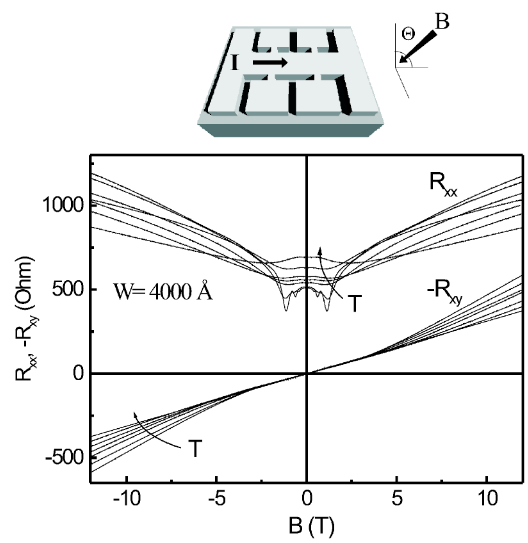

FIG. 2. Diagonal and Hall resistances of the 4000 Å wide parabolic well as a function of the magnetic field at tilt angle $\Theta \approx 89.1$ deg for different temperatures: 1.5 K, 4.2 K, 10 K, 15 K, 20 K, 30 K, 50 K. Top—geometry of the experiment.

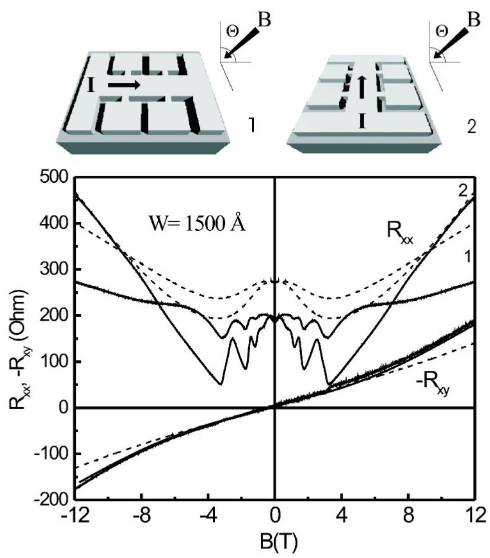

FIG. 3. The longitudinal and Hall resistance of the 1500 Å wide parabolic well as a function of the magnetic field at $\Theta \approx 89.6$ deg for different temperatures. Curves 1 (solid for $T=1.5$ K, dashed for $T=30$ K) show magnetoresistance, when the magnetic field is directed parallel to the current flow. Curves 2 (solid for $T=1.5$ K, dashed for $T=30$ K) show magnetoresistance, when the magnetic field is directed perpendicular to the current flow. Hall resistance is shown for $T=1.5$ K (solid) and $T=30$ K (dashes). Top—geometry of the experiment.

increases. Such anomalous behavior is observed only at a low temperature, at $T=30-50$ K, R_{xy} is recovered and demonstrates ordinary behavior. Figure 3 shows the magnetoresistance and Hall effect for a 1500 Å-wide parabolic well for two different temperatures and orientations of the current and magnetic field. In this samples we have 4 subbands occupied, as we may see from Shubnikov de Haas oscillations at a low magnetic field. Figure 3 shows also that the amplitude of the magneto-oscillations for geometry, when the magnetic field was directed parallel to the current flow (curves 2), is smaller than for the magnetic field directed perpendicular to the current (curves 1), in accordance with theory for 3D SdH oscillations.[7] Hall resistance in this sample at high magnetic field $B>4$ T also deviates from a low-field slope and becomes temperature dependent. It is worth noting that this anomalous behavior does not depend on the orientation of the magnetic field and current flow. Indeed, we check it for 1500 and 4000 Å-wide parabolic wells and found similar behavior. Figure 4(a) shows the Hall resistance for 4000 Å-wide parabolic well in details. At relatively high ($T=30-50$ K) temperatures in a quasiparallel magnetic field we observed a linear Hall effect (curves 1), described by a conventional equation,

$$R_{xy}/\cos\Theta = -B/en_s, \tag{1}$$

where e is the electron charge. At low ($T=1.5$ K) temperature we found that for a magnetic field B below 4.2 T Hall resistance is not changed, however, for $B>4.2$ T it demonstrates unusual behavior [curve 2 in Fig. 4(a)], which may be described by the equation

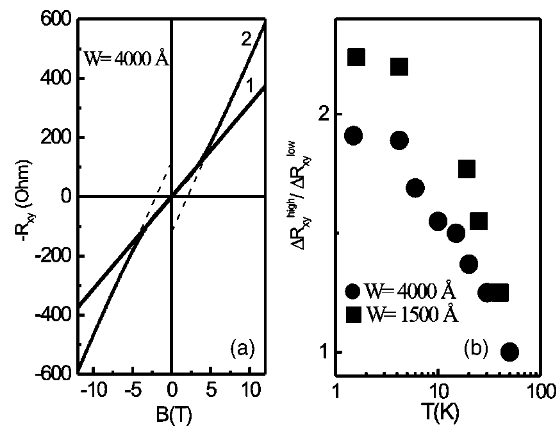

FIG. 4. The Hall resistance of a 4000 Å-wide parabolic well at $\Theta \approx 89.1$ deg as a function of the magnetic field at $T=50$ K (1) and $T=1.5$ K (2). Dashes show Eq. (2). (b) The ratio between the slope of the Hall resistance at a low and high magnetic field as a function of temperature for two samples.

$$R_{xy}/\cos\Theta = -A\times(B-B_0)/en_s, \tag{2}$$

where $B_0=2$ T, and A is the temperature-dependent coefficient. The Hall slope $R_H=\Delta R_{xy}/\Delta B$ gradually increases, when the temperature decreases, and becomes 2 times larger at $T=1.5$ K than at a low field and high temperatures [Fig. 4(b)].

Figure 5 shows experimental traces of the diagonal and Hall resistances for samples with different width W. All samples were illuminated. It is worth noting that the bulk Fermi energy in our system decreases with the width as $E_F=\hbar^2(3\pi^2 n_+)^{2/3}/(2mW^{4/3})$ and does not depend on the electron sheet density. Since the characteristic energy of the square well $E_0=(2\pi)^2\hbar^2/(8mW^2)$ decreases with the width

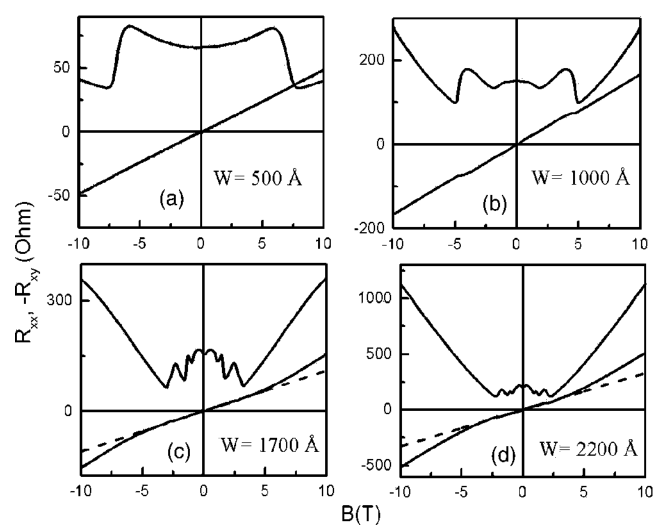

FIG. 5. The longitudinal and Hall resistance as a function of the magnetic field for various parabolic wells: (a) 500 Å ($n_s=6.4\times10^{11}$ cm^{-2}, $\Theta\approx89.7$ deg); (b) 1000 Å ($n_s=6.0\times10^{11}$ cm^{-2}, $\Theta\approx89$ deg); (c) 1700 Å ($n_s=5.0\times10^{11}$ cm^{-2}, $\Theta\approx89.4$ deg); (d) 2200 Å ($n_s=5.0\times10^{11}$ cm^{-2}, $\Theta\approx88.6$ deg), $T=1.6$ K. Dashes show Eq. (1).

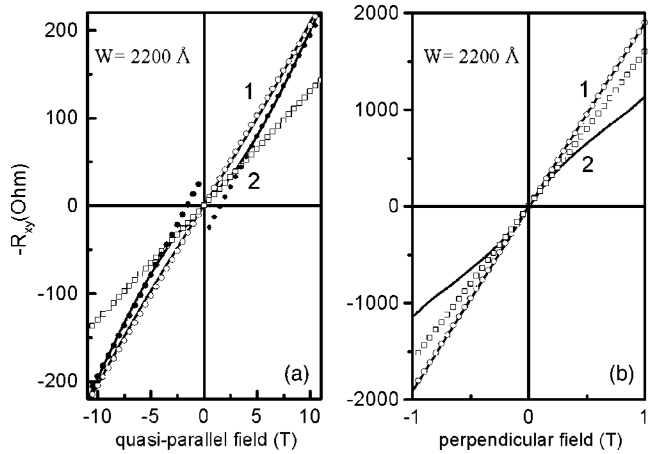

FIG. 6. The Hall resistance of a 2200 Å-wide parabolic well in quasiparallel ($\Theta \approx 89.4$ deg) (a) and perpendicular (b) magnetic fields, $T=4.2$ K. The curves (1) and (2) are measured before and after illumination (see Table I). Open circles and squares show Eq. (1); full circles show Eq. (2).

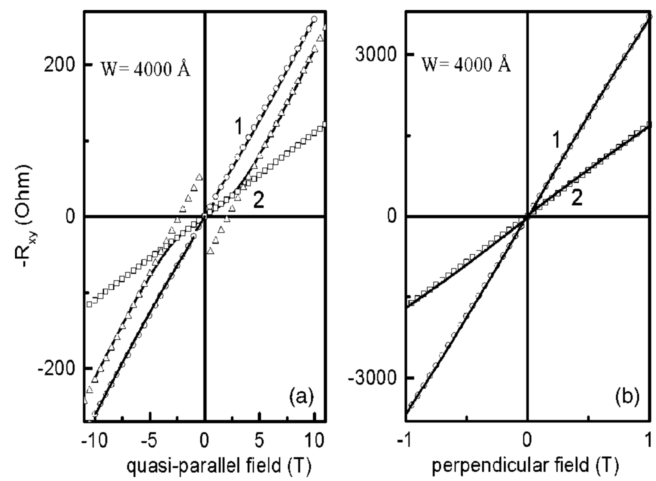

FIG. 7. The Hall resistance of a 4000 Å-wide parabolic well in quasiparallel ($\Theta \approx 89.4$ deg) (a) and perpendicular (b) magnetic fields, $T=4.2$ K. The curves (1) and (2) are measured before and after illumination (see Table I). Open circles and squares show Eq. (1), triangles show Eq. (2).

more rapidly than the Fermi energy, we have 2, 2, 3 and 4 subbands for 500, 1000, 1700 and 2200 Å parabolic well, consequently. Magnetoresistance oscillations in Fig. 5 are resulting from the depopulation of theses subbands in a quasiparallel magnetic field. We may see that the Hall slope change ΔR_H is observed only in wide parabolic wells with width $W > 1500$ Å. Note that in the PQW with a width smaller than 1000 Å electrons occupy only central part of the well and the ratio $f = W_{eff}/W$ is less than 0.3 even after illumination.

As we already mentioned previously, the width of the electronic slab increases after illumination. This particular property of the parabolic well makes it suitable for the practical realization of the electronic devices based on the manipulation of the average g factor. We checked the dependence of the Hall slope R_H in the perpendicular and quasiparallel magnetic field on the electron density. Figures 6 and 7 show such Hall slopes for two samples with different width W before and after strong illumination. We may see that the Hall effect is linear before illumination for both samples and field configurations. After illumination the behavior of the Hall resistance in a quasiparallel field is dramatically changed: the low field slope decreases in accordance with Eq. (1), which corresponds to an increase of the electron sheet density, however, at $B > 4.2$ T the Hall slope is changed and described by Eq. (2). In a perpendicular magnetic field we observed the conventional behavior of the Hall resistance for a 4000 Å width well. For a 2200 Å width parabolic well the Hall slope decreases at a high field. We will discuss the Hall slope change in the perpendicular field in the next section. Here we only emphasize the difference in the Hall slope behavior in quasiparallel and perpendicular magnetic fields. Such an observation justifies that increase of R_H, which is shown in Figs. 2–4, is due to the presence of the strong in-plane magnetic field.

Figure 8 shows the Hall effect for a 1500 Å width parabolic well after illumination for different tilt angles. We may see that the Hall slope change is observed only in a quasiparallel magnetic field. Indeed we checked that the angle is not changed during the magnetic field sweep, since even a small angle variation ($\theta \sim 0.1$ deg) may lead to a 1.6 times change in the Hall slope. For this aim we measure the Hall resistance in the narrow square GaAs quantum well and heterostructuras $Ga_{1-c}Al_cAs/GaAs$ during the same sweep of the magnetic field and find the conventional linear behavior. We also measure PQW in an almost parallel magnetic field, when Hall resistance approaches ~ 10 Ohms, which corresponds to the tilt angle ~ 89.97 deg. For this tilt angle the same angle variation $\theta \sim 0.1$ deg leads to 4.3 times change in the Hall slope. We do not find this large R_H variation, and the behavior of the Hall slopes is similar to behavior at $\Theta \approx 89.7$ deg, however, Hall traces are much more noisy and less reliable. We also believe that density and width depen-

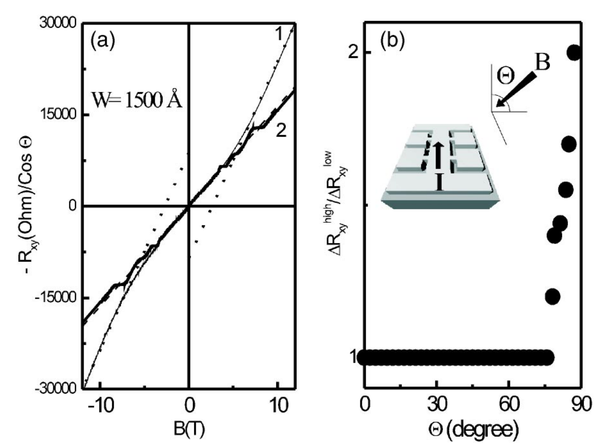

FIG. 8. Normalized Hall resistance at $\Theta \approx 89.6$ deg (1) and at $\Theta = 0$ (solid curve 2) as a function of the magnetic field, $T=1.5$ K. Dashes are the normalized Hall resistance at $\Theta \approx 89.6$ deg as a function of the magnetic field at $T=50$ K. Dots show Eq. (2). (b) The ratio between the slope of the Hall resistance at the low and high magnetic field as a function of tilt angle Θ.

dence measurements, which are shown in Figs. 5–7 justify that the angle is not changed during the magnetic field sweep.

III. THE HALL EFFECT IN A QUASI-TWO-DIMENSIONAL SYSTEM WITH SEVERAL SUBBANDS

It is well known that the presence of multiple carrier types (i.e., in the different subbands) with different mobilities causes changes in the Hall coefficient. In the case of the two subbands populated, the Hall resistance is given by the formula[8]

$$R_{xy} = -\frac{\langle \mu^2 \rangle + (r\mu_1\mu_2 B)^2}{\langle \mu \rangle^2 + (r\mu_1\mu_2 B)^2}\frac{B}{en_s}, \tag{3}$$

where $\mu_i = e\tau_i/em^*$ $(i=1,2)$ is the subband mobility, τ_i is the transport scattering time, r is a factor depending on the intersubband scattering, which is 1, for two independent conduction channels. The average mobility is defined as

$$\langle \mu \rangle = \frac{n_1\mu_1 + n_2\mu_2}{n_1 + n_2}, \tag{4}$$

where n_i is the subband density $(i=1,2)$, with a similar definition of $\langle \mu^2 \rangle$. It is worth determining the limiting values for the Hall slope R_H:

$$R_H(0) = -\frac{\langle \mu^2 \rangle}{\langle \mu \rangle^2}\frac{1}{en_s} \tag{5}$$

and

$$R_H(\infty) = -\frac{1}{en_s}. \tag{6}$$

Since $\langle \mu^2 \rangle > \langle \mu \rangle^2$, the Hall slope in the low field is larger than the Hall slope in a strong magnetic field. It is valid for any value of factor r, however, for $r<1$ the crossover from low to high field behavior may occur at a higher magnetic field than for $r=1$. In the quantum wells with three subband occupancy the situation is more complicated. We obtained the following equation for Hall resistance:

$$R_{xy} = -\frac{c_1 + c_2B^2 + c_3B^4}{c_4 + c_5B^2 + c_3B^4}\frac{B}{en_s}, \tag{7}$$

where c_i are parameters depending on the mobility and density of electrons in the each subband. Indeed we find that the Hall slope in low field $R_H(0)$ is larger than the Hall slope in strong magnetic field $R_H(\infty) = -1/en_s$ as in the case of the two subband occupancy. We may conclude here that it is valid for all quantum wells with multiple subband occupancy.

Let us focus on the experimental data. Figures 6(b) and 7(b) show the Hall resistance in a low perpendicular magnetic field for different electron densities and geometrical width. We find that after illumination of a 2200 Å-wide parabolic well the third subband in this sample becomes populated. Before illumination this parabolic well has only two occupied subbands, and we may see in Fig. 6(b) that $R_H(0) = R_H(\infty)$. When the third subband starts to become occupied, the Hall slope in a low field is changed and we obtain $R_H(0) > R_H(\infty)$ in accordance with Eq. (6). In principle we can fit the data in Fig. 6(b), magnetoresistance data and zero field conductivity, and deduce transport mobilities for each subbands. The electron sheet density for each subband is extracted from an analysis of the Shubnikov de Haas oscillations frequency. The detailed analysis of the magnetotransport results in quantum well with three subband occupancy will be done in a forthcoming publication. It should be emphasized here that the behavior of the Hall resistance in a quasiparallel magnetic field is completely different from $R_{xy}(B)$ in a perpendicular magnetic field. It is clear that the Hall slope in a perpendicular field $R_H(0)^{\perp} > R_H(\infty)^{\perp}$, on the other hand, in the quasiparallel magnetic field $R_H(0)^{\parallel} < R_H(\infty)^{\parallel}$. Naively, the opposed B dependence of the Hall slope was expected. First, the in-plane field leads to the depopulation of the subbands, therefore at the field corresponding to the onset of the depopulation of the last subband ($B_c > 2.5$ T for a 2200 Å wide well) conventional Hall slope $R_H(\infty)^{\parallel} = -1/en_s$ should be observed. Second, as in the perpendicular field case, the crossover from low to high field behavior leads to the change of the Hall slope from $R_H(0)$ to $R_H(\infty) = -1/en_s$. However, this crossover depends on the perpendicular component of magnetic field $B_{\perp}$, and in accordance with Fig. 6(b) occurs at $B_{\perp} = 0.2$ T. This value corresponds to $B_0 = 20$ T for tilt angle $\Theta \approx 89.4$ deg, which is much larger than the crossover magnetic field $B_0 = 2$ T obtained in the experiment [Fig. 6(a)]. Therefore we may conclude here that the changes in the Hall slope due to multiple subband occupancy and subband depopulation cannot provide a conventional explanation for our observation in quasiparallel magnetic field.

Note that in Fig. 6(a) $R_H(0)^{\parallel} = R_H(\infty)^{\perp} = -1/en_s$. We expect that for a quantum well with multiple subband occupancy in the low field $R_H(0)^{\parallel} = R_H(0)^{\perp}$, which disagrees with our observations. It may be explained by the fast depopulation of the third subband, which already occurs in a low field, and in the interval $0.2\ \text{T} < B < 2.5$ T only two subbands (or one) are occupied. Since for two subband occupancy we find $R_H(0) = R_H(\infty)$ [see Fig. 6(b), curve 1], therefore the low field Hall slope in a quasiparallel magnetic field is determined by the conventional formula $R_H(0)^{\parallel} = -1/en_s$.

Figure 7 shows the Hall resistance for a 4000 Å wide well, which has 4 subband occupancy before illumination and 7–8 occupancy after illumination. We may see that in a perpendicular field the Hall slope is not changed with the magnetic field increase. It seems reasonable, because in the three-dimensional limit we expect the conventional value of the Hall slope $R_H = -1/en_s$ for all intervals of the magnetic field. In a quasiparallel magnetic field we find $R_H(0)^{\parallel} < R_H(\infty)^{\parallel}$ and $R_H(0)^{\parallel} = R_H(0)^{\perp} = -1/en_s$. Again, as for a 2200 Å-wide parabolic well, we cannot explain this effect by multiple subband occupancy and subband depopulation in a quasiparallel magnetic field.

IV. SPIN-DEPENDENT TRANSPORT IN A WIDE PARABOLIC WELL

We attribute the Hall slope change to the unusual spin properties of our system. This effect is different from the extraordinary Hall effect discovered in ferromagnets almost 50 years ago.[9] It was found that the Hall resistivity in ferromagnets is larger than in nonmagnetic metals and can be fitted empirically by the formula $R_{xy}=R_0B+R_SM$, where B is applied magnetic field, R_0 is the ordinary Hall coefficient, and R_S is the anomalous Hall coefficient. The magnitude of the anomalous Hall coefficient depends on the various parameters of the material and its structure. For example, the magnitude of R_S is extremely large in amorphous ferromagnetics, it can be larger than R_0 by a factor of a 100 to a 1000, and may have the opposite sign. The asymmetric scattering of electrons by magnetic atoms may lead to the anomalous Hall voltage in ferromagnetic samples.

It is now accepted that two main mechanisms are responsible for such an effect: the skew-scattering proposed in the work of Ref. 9, and the side jump proposed in Ref. 10. We should mention that in these models the carriers are assumed to be magnetic and the scattering centers nonmagnetic. Indeed the result should be the same, when the situation is reversed. For a skew-scattering model the plane wave is scattered by impurity in the presence of the spin-orbit coupling. In such a situation the amplitude of the wave packet becomes anisotropic and depends on the spin. The average trajectory of an electron is deflected by a spin-dependent angle, which is typically of order 10^{-2} rad. For the side jump mechanism the center of the wave packet is shifted during the scattering, and this shift is also spin dependent. The typical lateral displacement during this process is 10^{-13} cm. It is worth noting that the deflection of the electron trajectory is very small, however, we can explain the large magnitude of R_S in an amorphous ferromagnet by a large number of scatterers, since practically every atom scatters the electron. Until recently, the anomalous Hall effect has been intensively studied only in magnetic structures.[11] In nonmagnetic semiconductors the anomalous Hall effect is very small, and can be separated from the ordinary Hall effect by magnetic resonance of the conduction electrons.[12] In InSb the Hall angle was found to be 3×10^{-4} rad, which is 4 orders of magnitude smaller than the angle for the normal Hall effect.

In our parabolic well the Hall slope change is completely different from the anomalous Hall effect. First, it has the same magnitude as an ordinary Hall effect. Second, in parabolic wells the dependence of the Hall resistance on a magnetic field is different from the Hall effect in ferromagnets: the slope of the Hall resistance at a higher field is larger than the slope at low B and depends on the temperature. In ferromagnets the slope at a higher field is determined by the ordinary Hall coefficient and usually it is smaller than an anomalous slope at low B.

We cannot ascribe the Hall effect in a parabolic well by the mechanism of the spin-dependent scattering described above, however, we suggest another explanation. As we already mentioned above, the effective g factor in such a structure changes the sign along the well width [see Fig. 1(a)]. In a strong quasiparallel magnetic field the magnetic length becomes smaller than the sample width. In this case the states in the different parts of the well along the z axis has different spin polarization: the center of the well is almost antialigned spin and the edge of the well is almost all aligned spins. It may lead to the suppression of the electron motion in the z direction in crossed electric and magnetic fields, since the motion in the z direction now requires the spin flip process. Such an effect is similar to the famous spin valve effect, which leads to the giant magnetoresistance in multilayers and has already found important applications. It should be emphasized here that the electron sheet density dependence or dependence of the effect on the width of the electronic slab just confirms our explanation. The magnitude of the Zeeman splitting in GaAs is -0.064 meV at $B=2.5$ T for electrons in the center of the sample. At the edge of the well the g factor is positive and the Zeeman energy has almost the same value, but is opposite in sign. Thus the spin flip process requires energy of 0.12 meV, which is comparable with temperature $T=1.6$ K, and therefore may lead to a significant spin-valve effect. For partially occupied parabolic wells with $f=W_{eff}/W<0.3$ ($W=500$ Å, 750 Å, 1000 Å and all samples before illumination) the g factor is varied between -0.2 and -0.44, and does not change the sign. Indeed the spin-valve effect is not expected in this case, in accordance with our observation.

We now turn to the more qualitative description of the Hall effect in our structures. The problem of quasi-two-dimensional electrons in a tilted magnetic field has been solved for a parabolic well by Meriln in his paper.[13] It is worth noting that spin splitting in the magnetic field was neglected.

The conductivity of electrons in a parabolic well in the presence of the tilted magnetic field is given by matrix

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ -\sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & -\sigma_{yz} & \sigma_{zz} \end{pmatrix}. \tag{8}$$

We consider the case when the magnetic field is tilted in the x direction. Before the calculation of the conductivities σ_{ij} we need to perform the following transformation of x and z coordinates: $x=X\cos\alpha-Z\sin\alpha$, $z=X\sin\alpha+Z\cos\alpha$. The angle of rotation α is different from the tilt angle Θ between the field and normal to the parabolic well plane z and is given by the formula

$$\tan(2\alpha)=(\hbar\Omega)(\hbar\omega_c)\sin\Theta/[(\hbar\Omega)^2-\ \hbar\omega_c)^2]. \tag{9}$$

In this new XyZ frame the energy of the electrons in a parabolic quantum well in a tilted magnetic field can be expressed analytically:[13]

$$E=\hbar\omega_1(n_\alpha+1/2)+\hbar\omega_2(n_\beta+1/2), \tag{10}$$

where

$$\omega_1=(\Omega_Z^2+\Omega^2\cos^2\alpha)^{1/2},$$

$$\omega_2=(\Omega_X^2+\Omega^2\cos^2\alpha)^{1/2},$$

$$\Omega_Z=\omega_Z\cos\alpha-\omega_X\sin\alpha,$$

$$\Omega_X=\omega_X\cos\alpha+\omega_Z\sin\alpha,$$

$\omega_c=eB/mc$, $\omega_Z=\omega_c\cos\Theta$, $\omega_X=\omega_c\sin\Theta$, and n_α and n_β are integers. Therefore we know the energy spectrum and electron wave functions in a parabolic well. With this knowledge it is possible to calculate transport coefficients employing the self-consistent Born approximation in an XyZ frame.[14] The conductivity tensor in this frame is given by the matrix

$$\sigma = \begin{pmatrix} \sigma'_{XX} & \sigma'_{Xy} & 0 \\ -\sigma'_{Xy} & \sigma'_{yy} & \sigma'_{yZ} \\ 0 & -\sigma'_{yZ} & \sigma'_{ZZ} \end{pmatrix}. \tag{11}$$

All conductivity components can be obtained analytically at zero temperature:[14]

$$\sigma'_{XX} = \frac{n_s e}{m} \frac{\gamma}{\omega_1^2 + \gamma^2}, \tag{12}$$

$$\sigma'_{ZZ} = \frac{n_s e}{m} \frac{\gamma}{\omega_2^2 + \gamma^2}, \tag{13}$$

$$\sigma'_{yy} = \frac{n_s e \Omega_Z^2}{m\omega_1^1} \frac{\gamma}{\omega_1^2 + \gamma^2} + \frac{n_s e \Omega_X^2}{m\omega_2^2} \frac{\gamma}{\omega_1^2 + \gamma^2}, \tag{14}$$

$$\sigma'_{Xy} = \frac{n_s e \Omega_Z}{m\omega_1^2} + \frac{\Omega_Z}{\omega_1} \int_{-\infty}^{\infty} \frac{df}{dE} \frac{\gamma(E)}{\omega_c} \sigma'_{XX}\, dE, \tag{15}$$

$$\sigma'_{yZ} = \frac{n_s e \Omega_X}{m\omega_2^2} + \frac{\Omega_X}{\omega_2} \int_{-\infty}^{\infty} \frac{df}{dE} \frac{\gamma(E)}{\omega_c} \sigma'_{ZZ}\, dE, \tag{16}$$

where $\gamma \approx (\pi\Gamma^2/2\omega_1 d)(\hbar/m\omega_2)^{1/2}$, Γ is level broadening, and d is the range of the electron-impurity interaction. The components of the conductivity tensor of matrix (3) can be obtained when we return back to the xyz frame,

$$\sigma_{xx} = \sigma'_{XX} \cos^2\alpha + \sigma'_{ZZ} \sin^2\alpha,$$

$$\sigma_{xy} = \sigma'_{Xy} \cos\alpha + \sigma'_{yZ} \sin\alpha,$$

$$\sigma_{xz} = (\sigma'_{Xy} - \sigma'_{yZ}) \cos\alpha \sin\alpha,$$

$$\sigma_{yz} = \sigma'_{yZ} \cos\alpha - \sigma'_{Xy} \sin\alpha,$$

$$\sigma_{zz} = \sigma'_{ZZ} \cos^2\alpha + \sigma'_{XX} \sin^2\alpha,$$

$$\sigma_{yy} = \sigma'_{yy}.$$

Finally, the Hall longitudinal and transverse resistivity components are obtained by inverting the conductivity tensor. For Hall resistivity we have

$$\rho_{xy} = -\frac{\sigma_{xy}\sigma_{zz} + \sigma_{xz}\sigma_{yz}}{\sigma_{xx}\sigma_{yy}\sigma_{zz} + \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{xz}^2 + \sigma_{zz}\sigma_{xy}^2 + 2\sigma_{xy}\sigma_{xz}\sigma_{yz}}. \tag{17}$$

In Fig. 6 we plot the dependence of the Hall resistivity on the magnetic field for $\hbar\Omega = 2.85$ meV, $\Theta \approx 89$ deg, $\Gamma = 0.8$ meV and $n_s = 3.2 \times 10^{11}$ cm^{-2}. Indeed we find that the Hall resistivity depends on the electron density and tilt angle, and is not sensitive to the confining potential. It is worth noting that the parameter Γ is deduced from the zero field conductivity

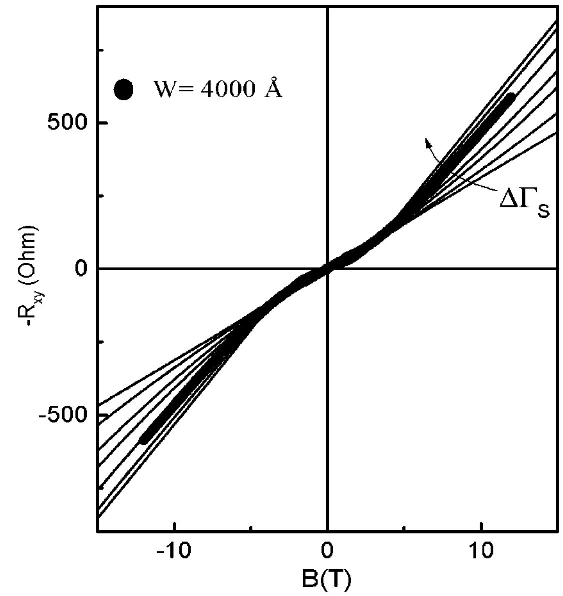

FIG. 9. The Hall resistance calculated from Eq. (12) for different parameters $\Delta\Gamma_S$ (meV): 0.0, 0.01, 0.03, 0.05, 0.1, 0.2, 0.3. Circles—experimental curve for a 4000 Å-wide parabolic well at $\Theta \approx 89$ deg and $T = 1.5$ K.

measurements. The range of the electron-impurity interaction is not well known in our structure, we take it to be 0.1–0.2 μm. Now we consider the electron motion in the z direction. As we already mentioned above, this motion is strongly suppressed, because it requires a spin flip process due to variation of the g factor along the z axis. We cannot calculate the spin flip process in our structure. Moreover, the electronic motion in the x direction is strongly coupled with that in the z direction. However, in the new coordinate system XyZ, the electronic motion in the X and Z directions is decoupled. In principle, in this case we may separate the scattering time for both directions, although spin flip processes indeed are included into the scattering time for both conductivity components σ'_{XX} and σ'_{ZZ}. We may introduce phenomenological parameter $\Delta\Gamma_S$, which is responsible for the spin-valve effect in the z direction. We denoted $\Delta\Gamma_S = \Gamma_{ZZ} - \Gamma_{XX}$, where Γ_{ZZ}, Γ_{XX} are levels broadening for X and Z directions, added it to the γ in Eq. (8), and calculated the Hall conductivities for various value of $\Delta\Gamma_S$. Figure 9 plots ρ_{xy} as a function of the magnetic field. As we expected, additional asymmetric (for X and Z directions) scattering leads to the increase in the slope of the Hall resistance. It is worth noting that at $B < 4$ T the Hall resistance is not linear and is lower than Hall resistance for the $\Delta\Gamma_S = 0$ case. This can be explained by simplifications that are done in our model. Spin flip scattering is strongly depends on the magnetic field, it turns on only after hybrid level depopulation, when a local g factor becomes z dependent. Since we are interested here in the high field slope of the Hall resistance, the curves plotted in Fig. 6 are calculated for B-independent $\Delta\Gamma_S$. We may see that the magnitude of $\Delta\Gamma_S$ decreases with a temperature increase, since at high temperature the fast spin-flip processes destroys the spin valve effect and electrons move in the z direction with the same probability as in the x and y directions. In spite of the indirect character of obtained information, the behavior of the parameter $\Delta\Gamma_S$ may be interesting, because the amount of interest to the spin relaxation in a low-dimensional system is only poorly supported by the experiments, especially from transport measurements. Indeed a further theoretical investigation of the transport in a parabolic well with a locally varied g factor is required for a detailed comparison with experiments.

V. CONCLUSION

We demonstrated that the variation of the g factor along the well width is responsible for the Hall slope change in a wide parabolic well in the presence of the strong in-plane magnetic field. We attribute such a large Hall slope to the spin valve effect in the z direction, which also suppresses the motion in the y direction. It may be justified by the several experimental observations. First, the Hall effect deviates from the ordinary slope at $B > 4$ T, when the magnetic length becomes smaller than the well width, and the local g factor turns to be the z dependent. In a lower field the g factor should be calculated by averaging the local g factor along the z axis: $\langle g\rangle = (1/W)\int_{-W/2}^{W/2} g(z)|\Psi(z)|^2\, dz$, where Ψ is the electron wave function; therefore the motion in the z direction is not spin dependent. Second, the Hall slope change is observed in a quasiparallel magnetic field, since the strong in-plane field makes the local g factor variable along the z axis. The change in the Hall slope also occurs in the presence of multiple carrier types as the subbands are depopulated, however, this change has an opposite sign and cannot explain our observation in the quasiparallel magnetic field. Finally, the Hall slope change is observed only in the parabolic well, which is almost completely filled by electrons. In these samples the effective g factor changes the sign across the well, which can lead to the spin-valve effect. In a partially filled parabolic well the sign change of the g factor along the z direction does not occur, and Hall resistance is not affected by spin-dependent transport. In order to quantitatively interpret the data we use a model that contains all the conductivity components of a parabolic well in a tilted field. A spin-valve effects lead to the anisotropy of these components, which can be a controlled by a phenomenological parameter.

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