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Spin Polarization by Tilted Magnetic Field in Wide $Ga_{1-x}Al_xAs$ Parabolic Quantum Wells

I. C. da Cunha Lima,[1] G. M. Gusev,[2] and J. R. Leite[2,3]

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Wide parabolic wells can be created by properly controlling the Al content during the growth of successive $Ga_{1-x}Al_xAs$ thin layers. Under a tilted magnetic field these systems present interesting transport properties, which are associated to their composition dependent *g*-factor. We present an exact solution for the eigenstates of an electron gas inside such a quantum well. We calculate the renormalized cyclotron frequencies as functions of the angle of tilt, as well as the density of states, and the Fermi level. We discuss the conditions for the existence of spin-polarized charge.

KEY WORDS: magnetic field; quantum wells; cyclotron frequency; *g*-factor.

1. INTRODUCTION

A harmonic confining potential in a semiconductor heterostructure can be created, for instance, by properly changing the Al concentration symmetrically in a wide $Ga_{1-x}Al_xAs$ layer. Since the first wide parabolic wells (WPW) grown by Sundaram *et al.* [1], and by Shayegan *et al.* [2], this system attracted much attention. More recently, WQWs under tilted magnetic field [3] have been used to study the transport properties in quantum Hall ferromagnets. In the particular case of $Ga_{1-x}Al_xAs$, besides the dependence of the gap and, in consequence, of the conduction band edge mismatch on the Al concentration, the *g*-factor also changes. Even its sign changes as the Al concentration increases above a certain value. This fact results in interesting properties, which appear in consequence of the mixing of the effects of the magnetic field associated to the kinetic part of the Hamiltonian, and those coming out of its interaction with the spin of the electron inside the WQW. In this work we show that an exact analytical solution can be obtained even in that case, by mapping the problem into the one solved by Merlin [4]. The tilted magnetic field, in association to the non-uniform *g*-factor results in spin-dependent renormalized Landau levels. We calculated the renormalized cyclotron frequency as a function of the angle of tilt and the strength of the applied magnetic field. We also calculate the density of states showing the conditions for the occurrence of a ferromagnetic state.

2. THEORY

A parabolic confining potential can be created in $Ga_{1-x}Al_xAs$ structure by growing successively thin layers with a proper Al composition [3]. Assuming as [0 0 1] the growth direction, and taking as $z = 0$ the position of a pure thin GaAs layer at the center of the structure, the Al composition of contiguous thin layers are made to vary quadratically with z, in both sides of the pure GaAs thin layer. Then, the mismatch of the bottom of the valence band changes quasi-continuously producing a confining potential which is approximately quadratic in z:

$$V_{\text{eff}}(z) = \frac{1}{2} m^* \omega_0^2 z^2. \tag{1}$$

Assuming that a profile of the Al composition $x(z) = a \cdot z^2$ is achieved, where a is a constant constant

[1]Instituto de Física, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, R.J., Brazil.
[2]Instituto de Física, Universidade de São Paulo, CP 66318, 05315–970, São Paulo, SP, Brazil.
[3]In memoriam.

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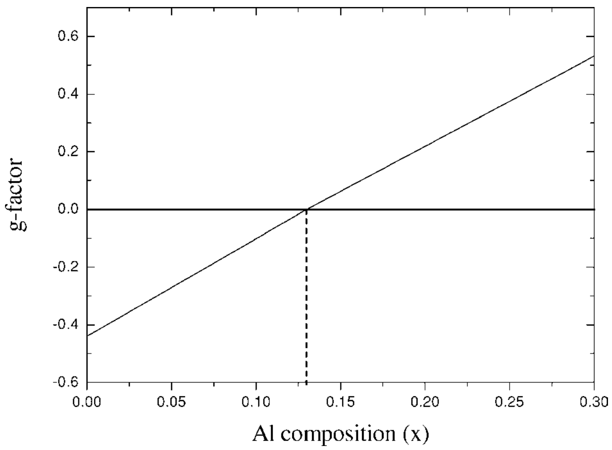

Fig. 1. Dependence of the g-factor with the Al composition in $Ga_{1-x}Al_xAs$ for $x \leq 0.3$.

determined by the growth process, the natural frequency ω_0 and and the constant a are related by the dependence of the conduction band mismatch with the Al composition. Samples have been grown in which the WQW width is 2000 Å, and the Al composition at the borders is $x = 0.29$. This makes $a = 2.9 \times 10^{-7}$ Å^{-2}.

It happens that the effective g-factor also changes with the Al composition. For instance, in pure GaAs it is $g_0 = -0.44$, in $Ga_{0.71}Al_{0.29}As$ it is 0.5, and the zero is reached in $Ga_{0.87}Al_{0.13}As$. Inside the range of interest, i.e., Al compositions below that giving the Γ–X transition of the minimum in the conduction band, as shown in Fig. 1, the dependence of the g-factor with the Al composition is approximately linear:

$$g(x) = g_0 + cx. \tag{2}$$

Between $x = 0$ and $x = 0.3$ we can take, then, $c = 3.24$. As a function of the z-position of the thin layer characterized by an homogeneous x, we obtain:

$$g(z) = g_0 + acz^2. \tag{3}$$

When a magnetic field $\boldsymbol{B} = (B_x, 0, B_z)$ is applied to an electron gas inside the WQW, the Hamiltonian becomes:

$$H = \frac{1}{2m}\left[(p_x + eB_z y)^2 + p_y^2 + (p_z - eB_x y)^2\right] + \frac{1}{2}m\omega_0^2 z^2 + g(z)\mu_B \vec{B}\cdot\vec{\sigma}, \tag{4}$$

where μ_B is the Bohr magneton, and where we have chosen the gauge $\boldsymbol{A} = y(-B_z, 0, B_x)$. Substituting Eq. (3) into Eq. (4), we have:

$$H = \frac{1}{2m}\left[(p_x + eB_z y)^2 + p_y^2 + (p_z - eB_x y)^2\right] + \frac{1}{2}m\Omega_\sigma^2 z^2 - g_0\mu_B \vec{B}\cdot\vec{\sigma}, \tag{5}$$

with

$$\Omega_\sigma = \sqrt{\omega_0^2 + \frac{ac\mu_B B}{m}\sigma}, \tag{6}$$

and $\sigma = \pm 1/2$. The total magnetic field couples to the electron spin introducing a rigid shift, as shown by the last term in Eq. (5), but it also renormalizes the natural frequency Ω_0. For anti-parallel alignment, the effect is to soften the harmonic potential till a critical value

$$B_c = \frac{m\omega_0^2}{ac\mu_B}, \tag{7}$$

At this field the electron with the anti-parallel spin polarization looses its 2-D character: the eigenstates become 3-D Landau levels, which are unaffected by the direction of the magnetic field. For higher fields, the parabolic well is renormalized into an effective parabolic barrier.

2.1. Field Perpendicular to the Plane

The absence of an x-component of the magnetic field allows an exact solution of the Schrödinger equation in real space. The component of the momentum in the x-direction is conserved, and the Hamiltonian in Eq. (5) reduces to:

$$H = \frac{p_y^2}{2m^*} + \frac{m^*\omega_c}{2}(y - y_0)^2 + \frac{p_z^2}{2m^*} + \frac{m^*\Omega_c}{2}z^2 - g_0\mu_B B\sigma. \tag{8}$$

This is the Hamiltonian of two uncoupled oscillator. The one corresponding to the motion in the y-direction is centered at $y_0 = \hbar k_x/eB$, that in the z-direction is centered at $z = 0$. The energy of the state $|n_a, n_b, k_x, \sigma\rangle$ is:

$$E_{n_a,n_b,\sigma} = \hbar\omega_c\left(n_a + \frac{1}{2}\right) + \hbar\Omega_c\left(n_b + \frac{1}{2}\right) - g_0\mu_B B\sigma. \tag{9}$$

These states are degenerate due to the value of k_x in the motion of the oscillator in the y-direction, the degeneracy being given by $eB/4\pi\hbar$.

2.2. Tilted Magnetic Field

With the renormalized frequency Ω_σ, Eq. (5) maps into the parabolic well problem with a tilted magnetic field, which was solved exactly by Merlin [4], mixing real and reciprocal spaces. Notice that p_x, the momentum x-component is, still, a constant of motion. It must be stressed the fact that the frequency is now spin-dependent, and depends on the strength of the applied magnetic field. Therefore, each spin polarization is submitted to its particular parabolic well. The Hamiltonian, again, can be decoupled into two independent harmonic oscillator, the spin degeneracy lifted, and the solution for the state $|n_a, n_b, k_x, \sigma\rangle$ is:

$$E^\sigma_{n_a,n_b} = \left(n_a + \frac{1}{2}\right) E^\sigma_a + \left(n_b + \frac{1}{2}\right) E^\sigma_b - g_0 \mu_B B\sigma, \tag{10}$$

where

$$E^\sigma_a = \left[\hbar^2\omega_c^2 \cos^2\alpha_\sigma + \hbar^2\Omega_\sigma^2 \sin^2\alpha_\sigma - \hbar^2\omega_c\Omega_\sigma \sin(2\alpha_\sigma)\sin(\theta)\right]^{1/2}, \tag{11}$$

$$E^\sigma_b = \left[\hbar^2\omega_c^2 \sin^2\alpha_\sigma + \hbar^2\Omega_\sigma^2 \cos^2\alpha_\sigma + \hbar^2\omega_c\Omega_\sigma \sin(2\alpha_\sigma)\sin(\theta)\right]^{1/2}, \tag{12}$$

with

$$\tan(2\alpha_\sigma) = 2\frac{\omega_c\Omega_\sigma \sin\theta}{\Omega_\sigma^2 - \omega_c^2}, \tag{13}$$

and

$$\sin\theta = \frac{B_x}{B}. \tag{14}$$

The cyclotron frequency ω_c is determined by the total magnetic field: $\omega_c = eB/m$. The degeneracy of the state whose energy is $E^\sigma_{n_a,n_b}$ is determined by the z-component of the magnetic field, $\pi e B_z/\hbar$, with the spin degeneracy now lifted.

We analyzed the case where the resonance is reached for a specific spin polarization: Let us assume, first, that for a specific spin polarization $\Omega_\sigma \to \omega_c + 0^+$. Then, $\tan(2\alpha_\sigma) \to +\infty, 2\alpha_\sigma \to \pi/2, \alpha_\sigma = \pi/4$. In that case, for this particular spin polarization in resonance,

$$E^\sigma_a = \hbar^2\omega_c^2(1 - \sin\theta)^{1/2} \tag{15}$$

$$E^\sigma_a = \hbar^2\omega_c^2(1 + \sin\theta)^{1/2} \tag{16}$$

On the other hand, if $\Omega_\sigma \to \omega_c + 0^-$, $\tan(2\alpha_\sigma) \to -\infty, 2\alpha_\sigma \to -\pi/2, \alpha_\sigma = -\pi/4$. The result will be the same, with the two modes, a, and b, interchanged.

3. RESULTS AND COMMENTS

Figure 2 shows the results for the eigenfrequencies $E_{a,b}$ as functions of the applied magnetic

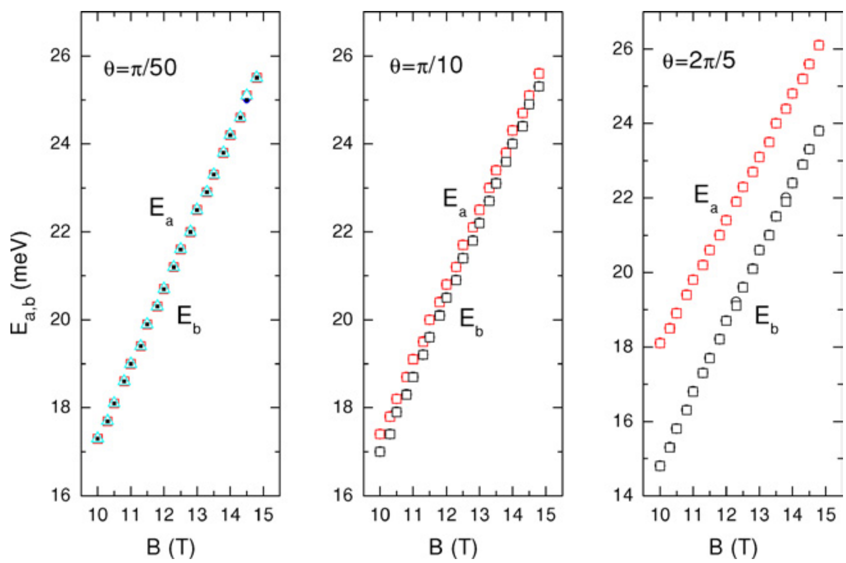

Fig. 2. Dependence of the renormalized natural frequencies $E^\sigma_{a,b}$ with the magnetic field: (a) $\theta = \pi/50$; (b) $\theta = \pi/10$; (c) $\theta = 2\pi/5$. In this scale differences between spin up and down are not visible.

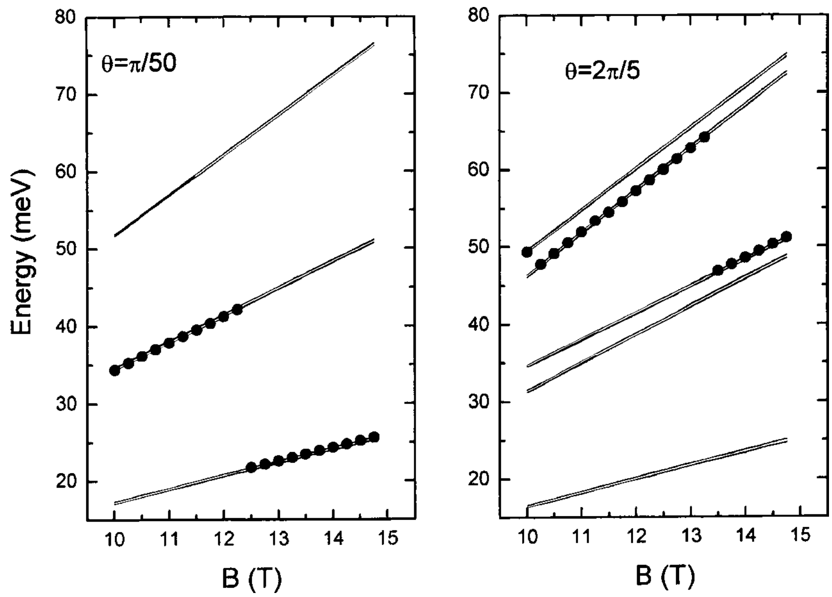

Fig. 3. Dependence of the Fermi level with the magnetic field for: (a) $\theta = \pi/50$; (b) $\theta = 2\pi/5$.

field. We present the case of tilt angles (a) $q = \pi/50$, (b) $\pi/10$, and (c) $2\pi/5$, corresponding to magnetic fields very close to the (xy) plane, low tilt angle, and nearly perpendicular to the plane. The calculation is performed for a carrier density (electrons) $n_s = 3. \times 10^{10}\,\mathrm{cm}^{-3}$, the value observed in the samples produced [3]. In the scale of the figure we cannot observe any difference between the spin polarized energies $E^{\mathrm{up}}_{a,b}$ and $E^{\mathrm{down}}_{a,b}$. However, there is a remarkable effect of the tilt angle. In almost planar field the two branches almost coincide, the difference remaining small at low angles. As the tilt angle approaches $\pi/2$ the two branches become distinguishable. Notice that the state degeneracy depends only on the z-component of the magnetic field. The limit $\theta = \pi/2$ maps into two independent oscillators with eigen-energies given Eq. (9).

Next we take the cases of (a) $\theta = \pi/50$ and (b) $\theta = 2\pi/5$ to show the dispersion curves as functions of the applied magnetic field, and how the Fermi level changes, in consequence. The results appear in Fig. 3. This figure is qualitatively similar to that of Landau levels in a two-dimensional electron gas. The difference is that now we have two independent modes per spin. As the magnetic field increases, the Fermi level jumps from one to another branch, with a specific spin-polarization. Therefore the spin polarization of the total charge changes with the magnetic field, and the density of states at the Fermi level has, for each value of the magnetic field, its specific spin polarization.

Despite the fact that the effective g-factor changes considerably inside the GaAlAs wide quantum well, even changing sign, and, in principle, we should imagine that each Larmor orbit would occur in regions corresponding to different coupling of the magnetic field with the electron spin, we have shown that the problem has an exact and simple solution. It corresponds to two uncoupled harmonic oscillators with spin-dependent natural frequencies, plus the coupling of the total magnetic field with the spin, this coupling being determined by the effective g-factor corresponding to the material at the center of the well. The simplicity of this result is important to understand the transport properties of such systems.

It is well known that in the quantum Hall regime electron–electron interaction leads to fully aligned electron spins, in what is called quantum hall ferromagnetism (QHF). This phenomena has already been observed in a wide parabolic well in tilted magnetic field.[?] The present approach makes it easier to study the conditions for the occurrence of QHF in those systems by adding the electron-electron interaction in the uncoupled harmonic oscillators solution.

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REFERENCES

1. M. Sundaram, A. C. Gossard, J. H. English, and R. M. Westervelt, Superlattices Microstruct. **4,** 682 (1988).
2. M. Shayegan, T. Sajoto, M. Santos, and C. Silvestre, *Appl. Phys. Lett.* **53,** 791 (1988).
3. G. M. Gusev, A. A. Quivy, T. E. Lamas, J. R. Leite, A. K. Bakarov, A. I. Toporov, O. Estibals, and J. C. Portal, *Phys. Rev. B* **65,** 205316 (2002), and references therein.
4. R. Merlin, *Solid State Commun*. **64,** 99 (1987).
5. See, for instance, D. K. Ferry and S. M. Goodnick, *Transport in Nanostructures* (Cambridge University Press, Cambridge, 1997).
6. G. M. Gusev, A. A. Quivy, T. E. Lamas, J. R. Leite, O. Estibals, and J. C. Portal, *Phys. Rev. B* **67,** 155313 (2003).