QUANTUM SIMULATION

Monitoring and manipulating Higgs and Goldstone modes in a supersolid quantum gas

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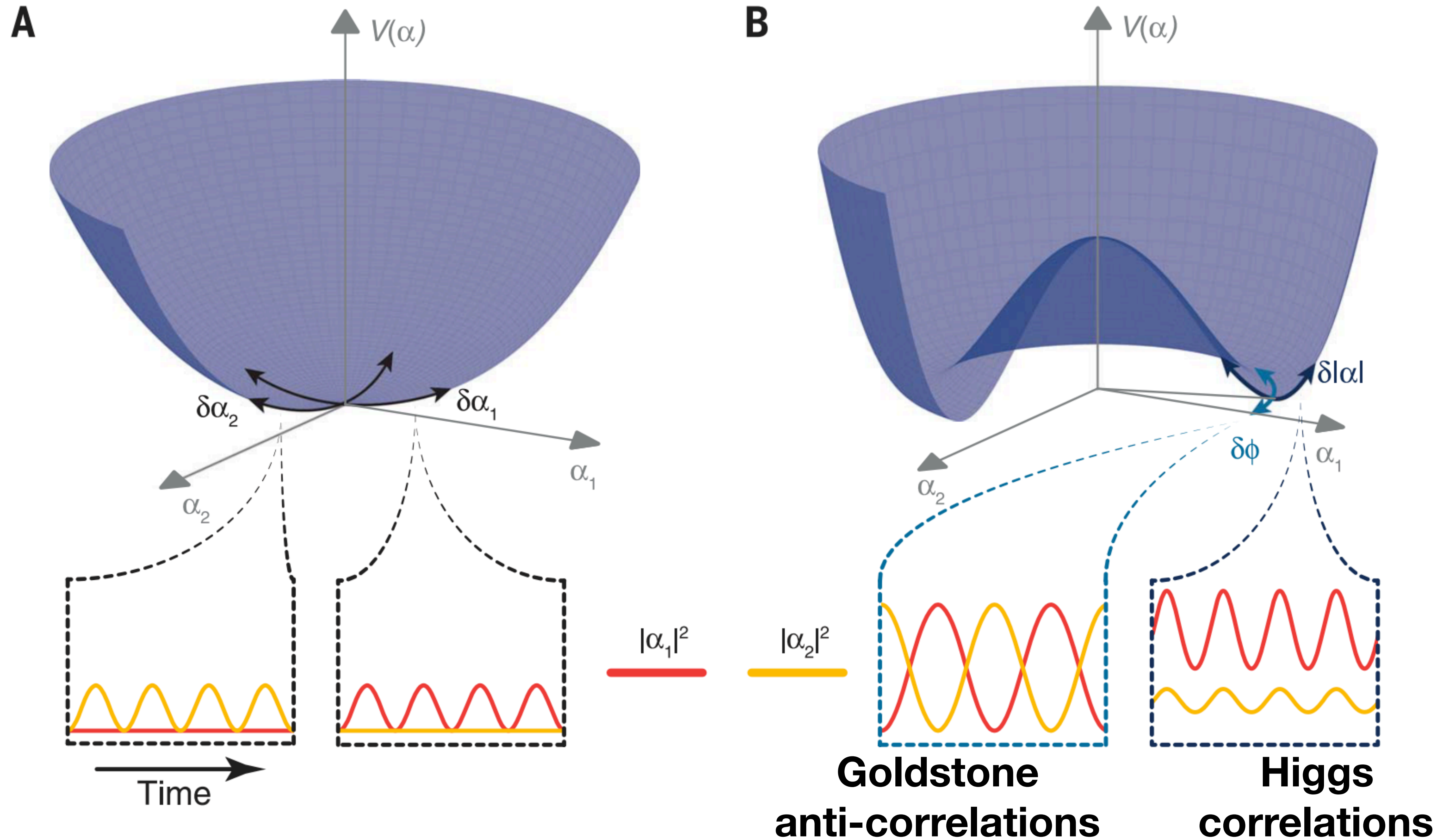
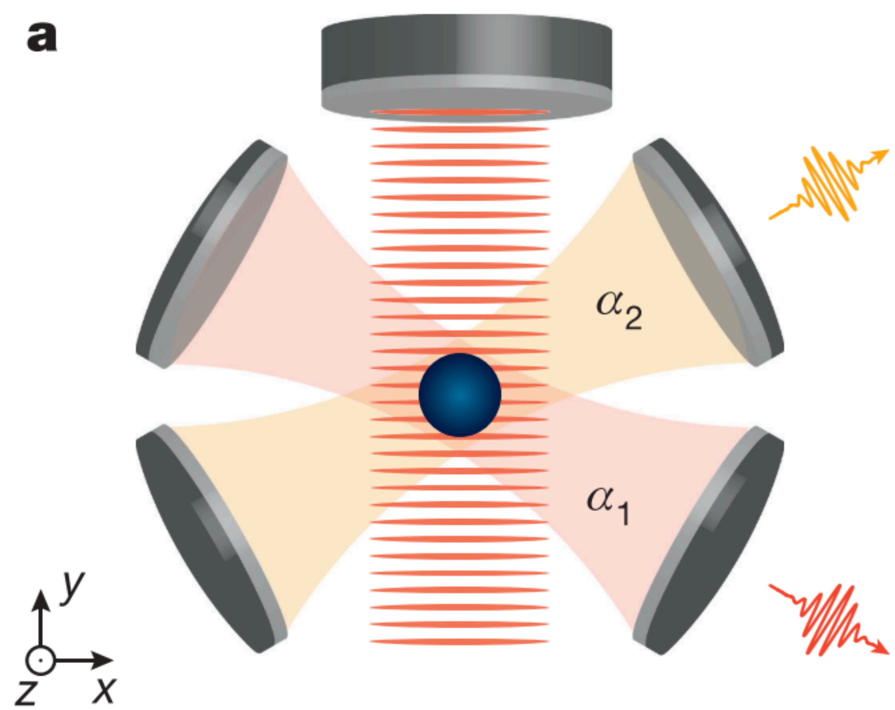
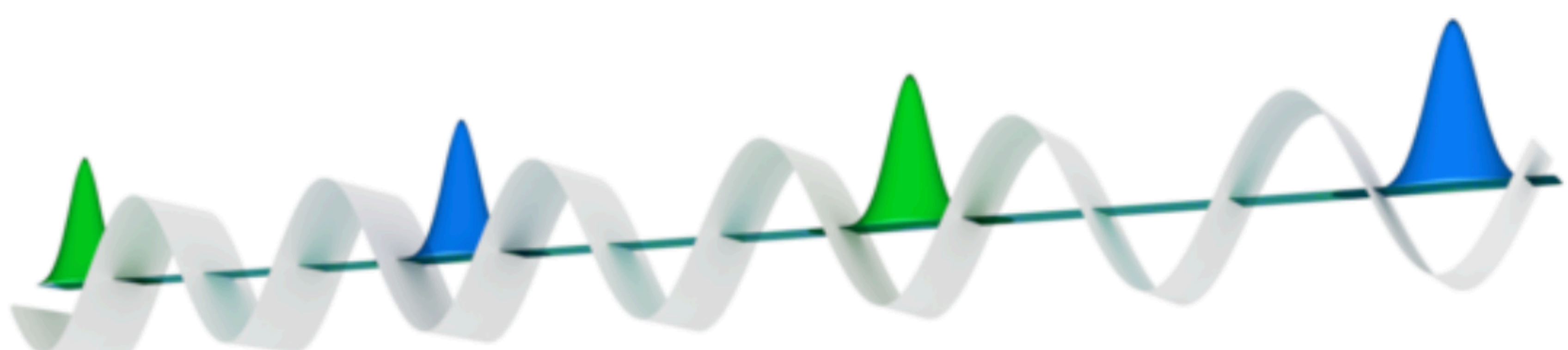


$$\alpha = \alpha_1 + i\alpha_2 = |\alpha|e^{i\phi}$$

$$V(\alpha) = r|\alpha|^2 + g|\alpha|^4$$


a
α2
α1
y
z
x



$$H_{sp} = -\Delta_1 a_1^\dagger a_1 - \Delta_2 a_2^\dagger a_2$$

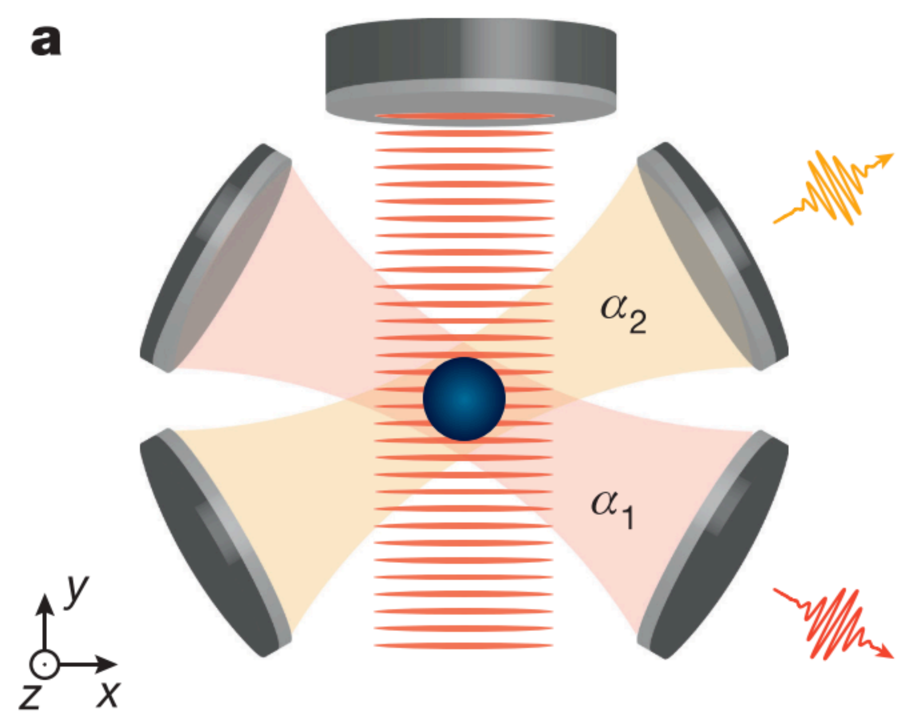
$$+\frac{p_x^2 + p_y^2}{2m} + \sum_{i=1,2} \Bigg\{ U_p \cos^2(\boldsymbol{k}_p \cdot \boldsymbol{r} + \phi) + U_i \cos^2(\boldsymbol{k}_i \cdot \boldsymbol{r}) a_i^\dagger a_i$$

$$+\eta_i (a_i^\dagger + a_i) \cos(\boldsymbol{k}_p \cdot \boldsymbol{r} + \phi) \cos(\boldsymbol{k}_i \cdot \boldsymbol{r}) \Bigg\}$$

$$\boldsymbol{k}_p = k\hat{y}$$

$$\boldsymbol{k}_1 = k\left[\hat{x}\sin(\pi/3) - \hat{y}\cos(\pi/3)\right]$$

$$\boldsymbol{k}_2 = k\left[\hat{x}\sin(\pi/3) + \hat{y}\cos(\pi/3)\right]$$



$$\mathcal{H} = \int dx dy \Bigg\{ \Psi^\dagger(\boldsymbol{r}) H_{sp} \Psi(\boldsymbol{r}) + g \Psi^\dagger(\boldsymbol{r}) \Psi^\dagger(\boldsymbol{r}) \Psi(\boldsymbol{r}) \Psi(\boldsymbol{r}) \Bigg\}$$

In this paper they neglect *g*

$$H_{sp} = -\Delta_1 a_1^\dagger a_1 - \Delta_2 a_2^\dagger a_2$$

$$+\frac{p_x^2 + p_y^2}{2m} + \sum_{i=1,2} \Big\{ U_p \cos^2(\boldsymbol{k}_p \cdot \boldsymbol{r} + \phi) + U_i \cos^2(\boldsymbol{k}_i \cdot \boldsymbol{r}) a_i^\dagger a_i$$

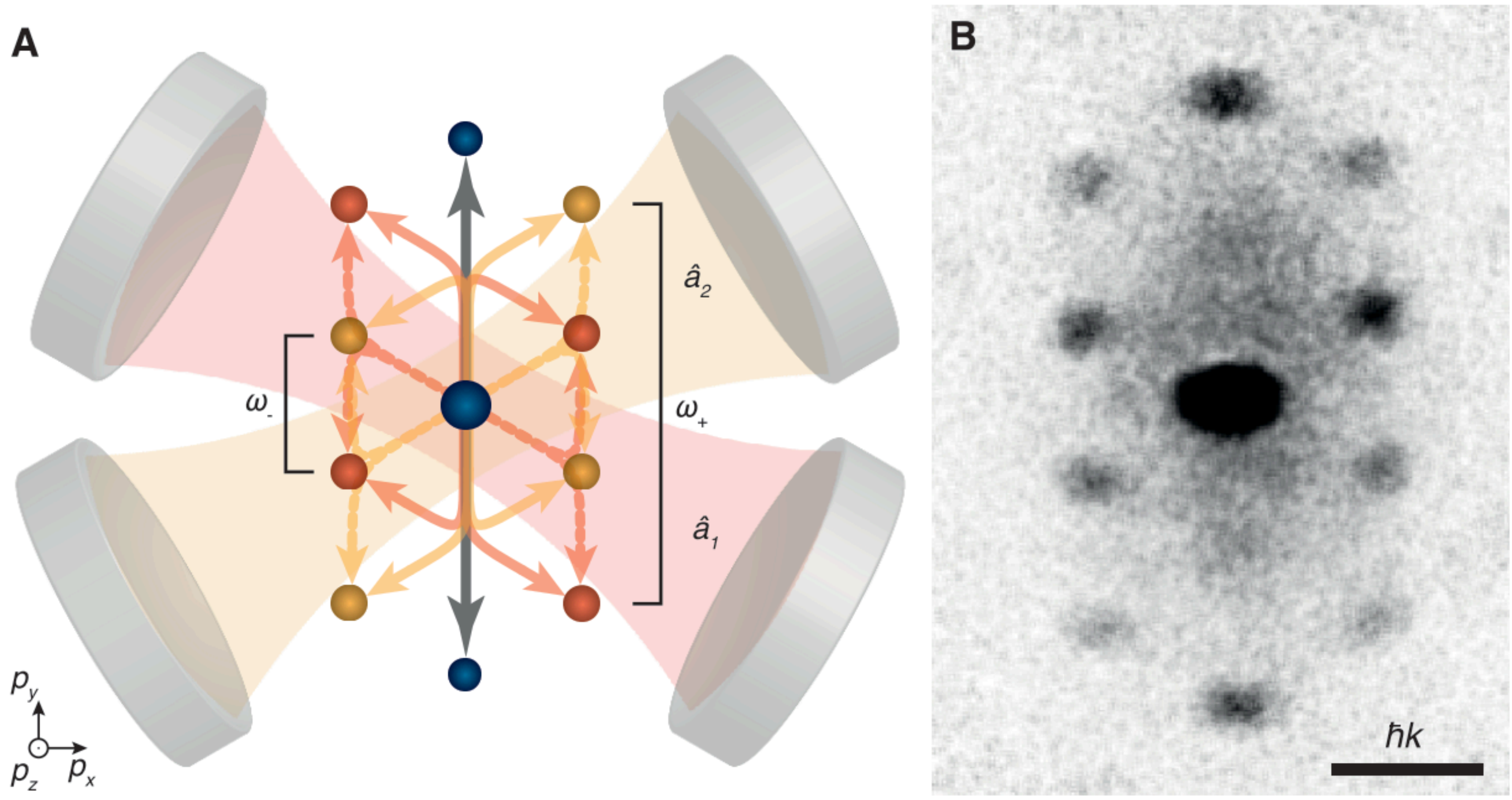
$$+\eta_i (a_i^\dagger + a_i) \cos(\boldsymbol{k}_p \cdot \boldsymbol{r} + \phi) \cos(\boldsymbol{k}_i \cdot \boldsymbol{r}) \Big\}$$

- The blue term has a special role:
  - Creation and annihilation of a cavity photon couples the BEC ground-state with the first 8 excited states

$$|\mathrm{gs}\rangle = |k_x = 0, k_y = 0\rangle$$

excited states = $| \pm \boldsymbol{k}_p \pm \boldsymbol{k}_i \rangle$

At low energies, the effective dynamics will involve only these 9 states.



Suppl. Fig. 1. **Atomic momentum states.** **(A)** Coherent scattering processes of pump photons into cavity 1 (red) or cavity 2 (orange) and back give rise to atomic momentum states at energies $\hbar\omega_- = \hbar\omega_{\text{rec}}$ and $\hbar\omega_+ = 3\hbar\omega_{\text{rec}}$. Pump photons can also be scattered back into the pump (gray). The coordinate system is with respect to momentum space. **(B)** Absorption image of the atoms in the supersolid phase after 25 ms ballistic expansion. All momentum states highlighted in **(A)** are populated.

Ansatz for the field operators

$$\Psi = \psi_0(\boldsymbol{r})c_0 + \sum_{i=1,2}\Big(\psi_{i-}(\boldsymbol{r})c_{i-} + \psi_{i+}(\boldsymbol{r})c_{i+}\Big)$$

Bosonic operators: $c^\dagger_{i,\pm}|0\rangle = |\boldsymbol{k}_p \pm \boldsymbol{k}_i\rangle$

With this reduced picture, the Hamiltonian becomes

$$H = \sum_{i=1,2}\Bigg\{ -\Delta_i a_i^\dagger a_i + \omega_+ c^\dagger_{i+}c_{i+} + \omega_- c^\dagger_{i-}c_{i-} + \frac{\lambda_i}{\sqrt{N}}(a_i^\dagger + a_i)\Big(c^\dagger_{i+}c_0 + c^\dagger_{i-}c_0 + \text{h.c.}\Big)\Bigg\}$$

$$\omega_+ = 3\omega_-$$

Augmenting the symmetry from Z2 to U(1)

$$H = \sum_{i=1,2} \Bigg\{ -\Delta_i a_i^\dagger a_i + \omega_+ c_{i+}^\dagger c_{i+} + \omega_- c_{i-}^\dagger c_{i-} + \frac{\lambda_i}{\sqrt{N}} (a_i^\dagger + a_i) \left(c_{i+}^\dagger c_0 + c_{i-}^\dagger c_0 + \text{h.c.} \right) \Bigg\} := H_1 + H_2$$

H1 and H2 both have a Z2 symmetry:

$$a_i \to -a_i$$
$$c_{i\pm} \to -c_{i\pm}$$

$$\Delta_1 = \Delta_2$$
$$\lambda_1 = \lambda_2$$

But the total Hamiltonian H is now invariant under a stronger U(1) symmetry

$$a_1 \to a_1 \cos\theta - a_2 \sin\theta$$
$$a_2 \to a_1 \sin\theta + a_2 \cos\theta$$
$$c_{1\pm} \to c_{1\pm} \cos\theta - c_{2\pm} \sin\theta$$
$$c_{2\pm} \to c_{2\pm} \sin\theta + c_{1\pm} \cos\theta$$

Generator

$$U = e^{-i\theta C}$$

$$C = -i \Big\{ a_1^\dagger a_2 - a_2^\dagger a_1 + \sum_{\sigma=\pm} (c_{1\sigma}^\dagger c_{2\sigma} - c_{2\sigma}^\dagger c_{1\sigma}) \Big\}$$

Order parameters

$$\alpha_i = \langle a_i \rangle$$

Two superfluid phases: $\alpha_1 \neq 0, \quad \alpha_2 = 0$ $\qquad$ $\lambda_i^{\mathrm{cr}} = \sqrt{-3\Delta_i \omega_- / 16}$

$\alpha_1 = 0, \quad \alpha_2 \neq 0$

Supersolid phase: $\alpha_1 \neq 0, \quad \alpha_2 \neq 0$

$$\lambda_1^{\mathrm{cr}} = \lambda_2^{\mathrm{cr}}$$

Supersolid = crystallization of a many-body systems + dissipation flow of the atoms

= breaking of 2 continuous symmetries: phase invariance of a superfluid and continuous translation invariance.

Holstein-Primakoff

They neglect the c+ term

$$H = \sum_{i=1,2} \Bigg\{ -\Delta_i a_i^\dagger a_i + \omega_+ c_{i+}^\dagger c_{i+} + \omega_- c_{i-}^\dagger c_{i-} + \frac{\lambda_i}{\sqrt{N}} (a_i^\dagger + a_i) \Big(c_{i+}^\dagger c_0 + c_{i-}^\dagger c_0 + \text{h.c.} \Big) \Bigg\}$$

$$a_i = \sqrt{N}\alpha_i + \delta a_i$$

$$c_{i-} = \sqrt{N}\psi_i + \delta c_{i-}$$

$$c_0 = \sqrt{N - \sum_{i=1,2} c_{i-}^\dagger c_{i-}}$$

Averages give the order parameters

$$\alpha_i, \ \psi_i$$

Fluctuation operators determine the excitation spectra

$$\delta a_i, \ \delta c_{i-}$$

(Gaussianization)

Normal phase

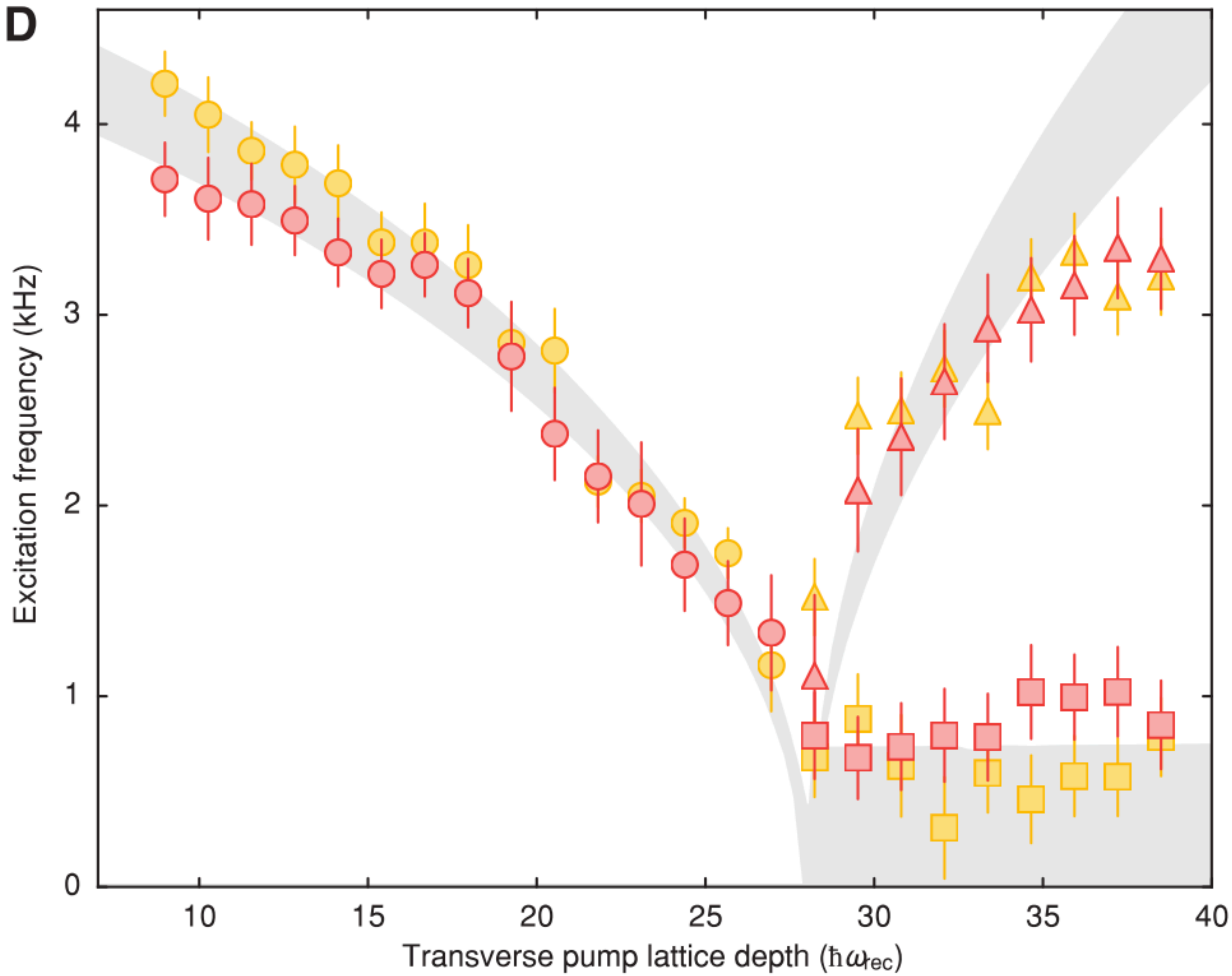
Two massive modes with frequency

$$\omega_i = \omega_- \sqrt{1 - \frac{\lambda^2}{\lambda_{\mathrm{cr}}^2}}$$

Supersolid phase

One mode is massive (complicated expression for the frequency): Higgs

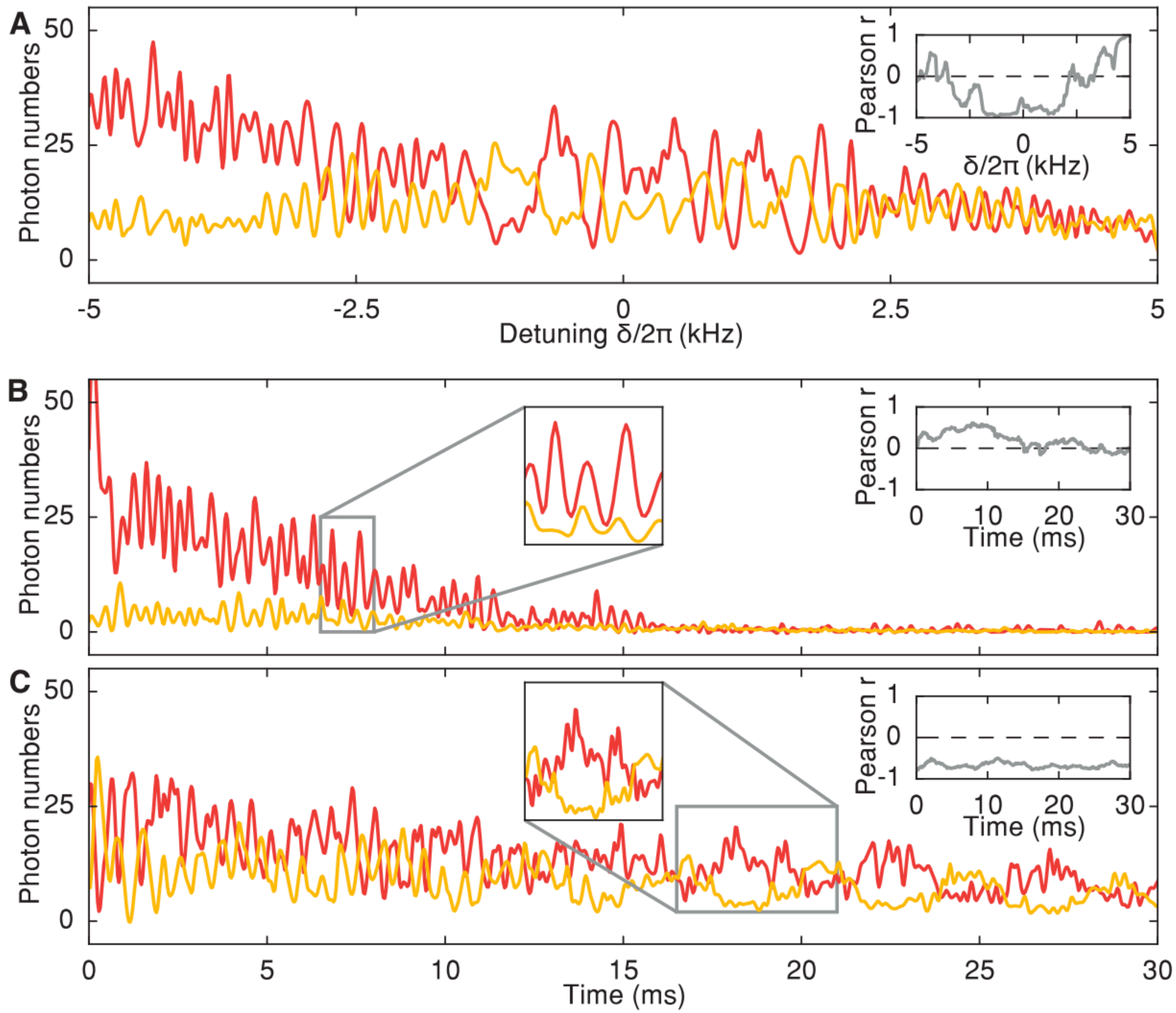
But the other mode is massless: Goldstone



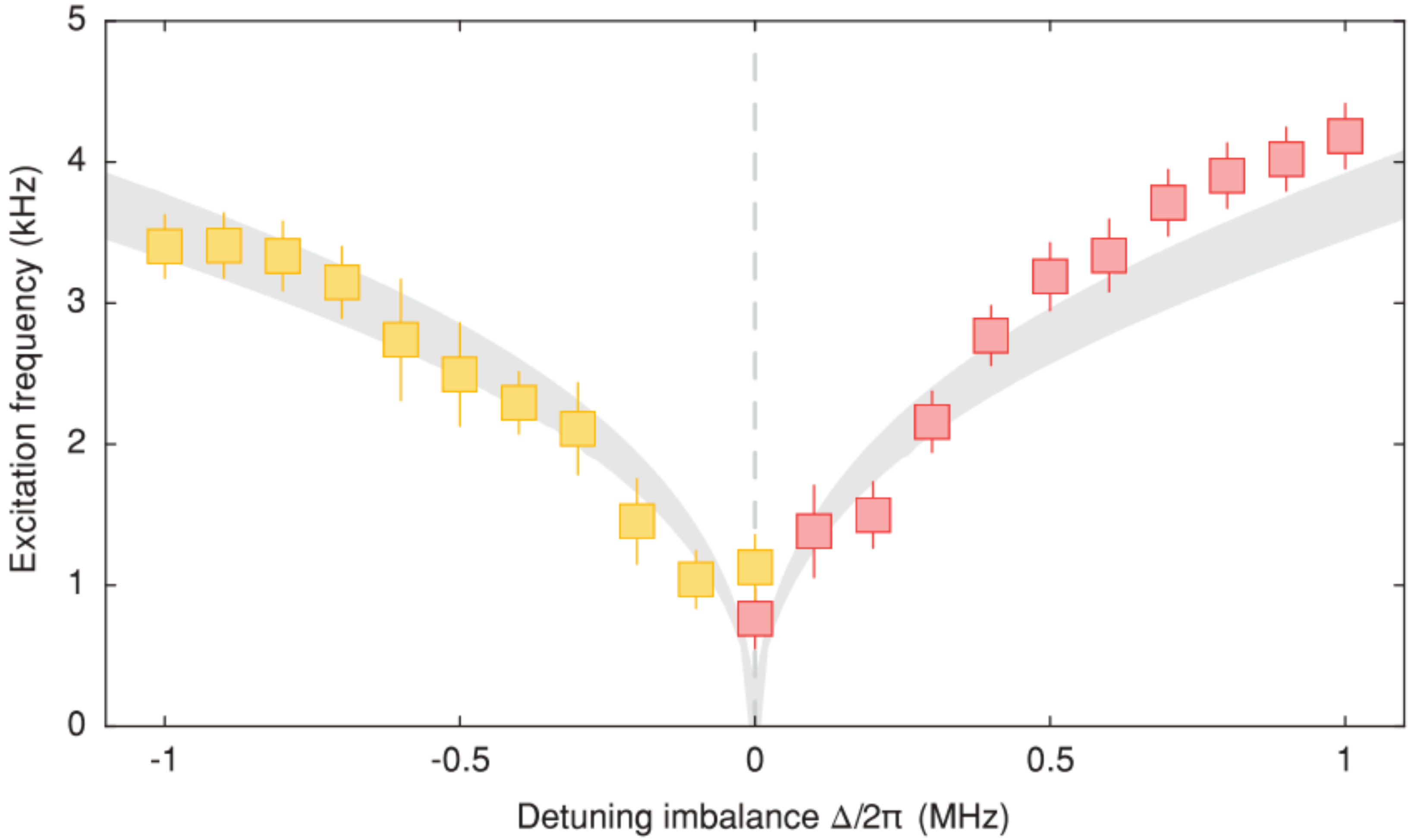
lattice depth “=“ λ

Probe the different modes by applying a time-dependent perturbation in the Hamiltonian which creates excitations above the BEC.

Dynamics of the Higgs and Goldstone modes



By explicitly breaking the U(1) symmetry they can make the Goldstone mode massive again.



Conclusions

- Real-time access to Higgs and Goldstone modes.
- Huge experimental achievement
- In my opinion what is coolest is:
    - Touches on properties of Higgs and Goldstone modes which are not usually discussed in theoretical papers, but are relevant from an experimental point of view.